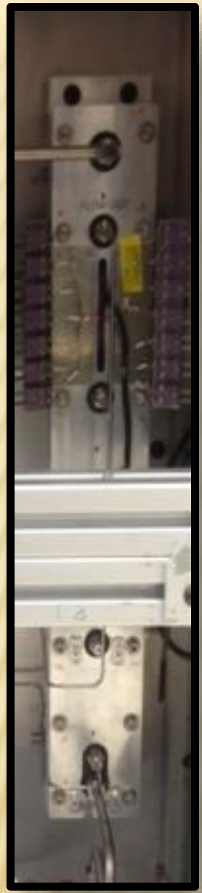

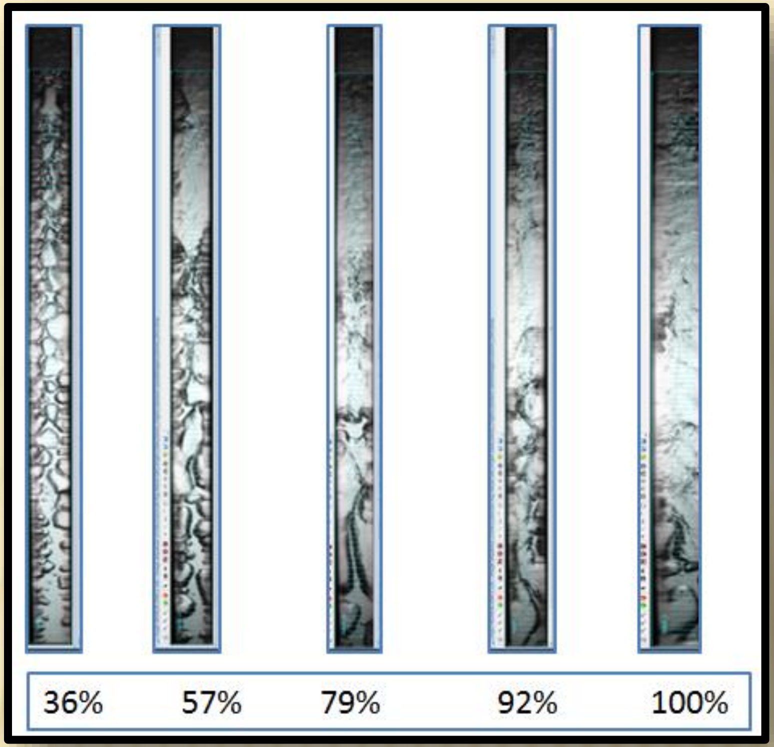

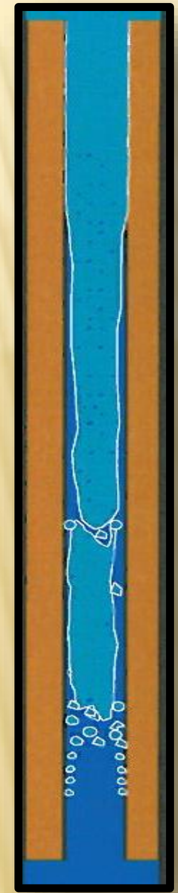

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TWO PHASE FLOWS AND HEAT TRANSFER

TFAWS- 2014, Cleveland OH
August 4-August 8, 2014

REFERENCES

References

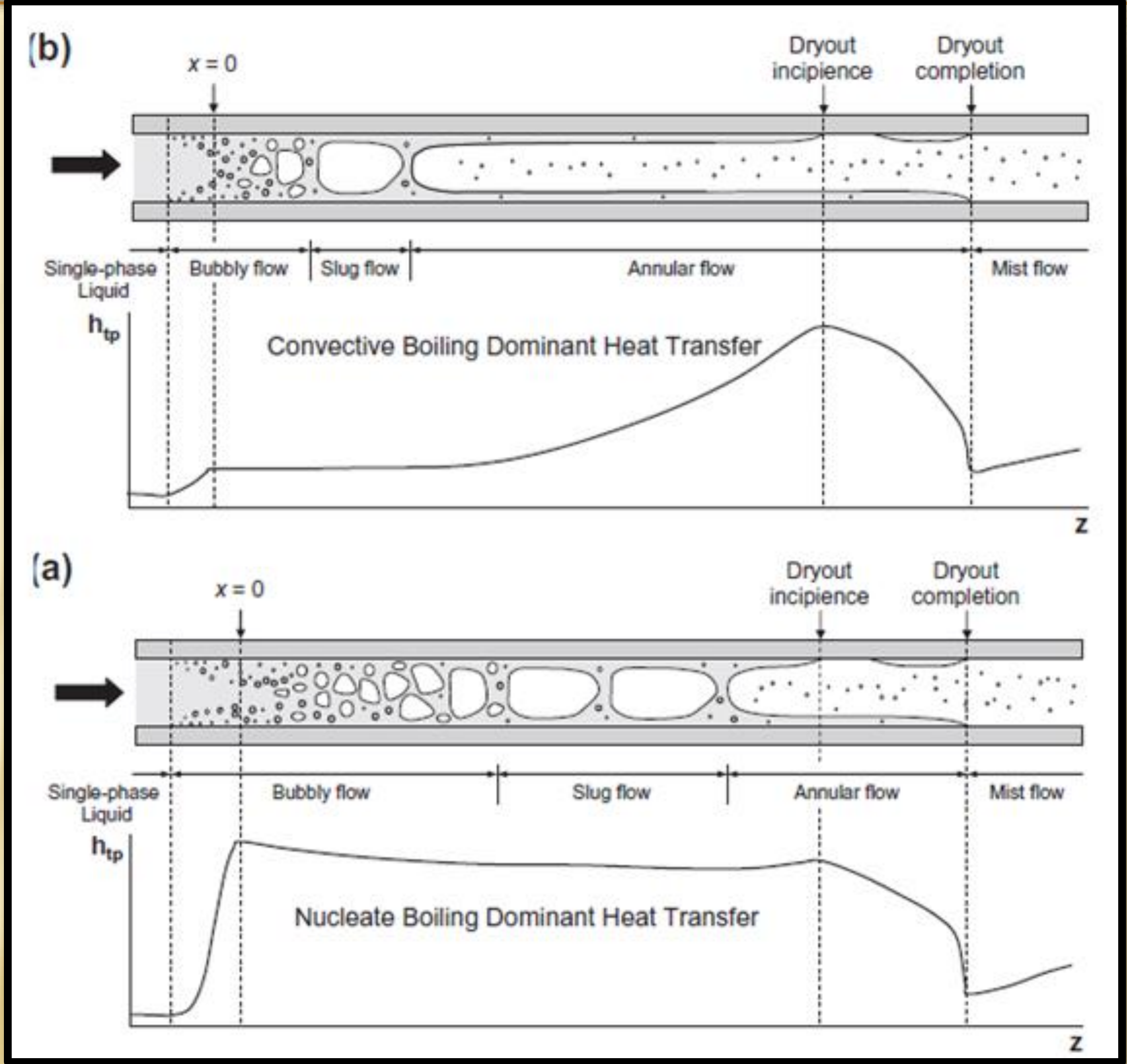
- 1. Mudawar I., "Two-Phase Flow and Heat Transfer", NASA-GRC, December, 2013, February, 2014
- 2. Collier J.G and Thome J.R, "Convective Boiling and Condensation", 3rd Edition, Oxford University Press Inc., NY, 2001

OUTLINE

- In this short course, we will address the following topics
  - Two-phase flows hydrodynamics and pressure drop of evaporating and condensing flows
    - Homogeneous Equilibrium Model
    - Separated Flow Model
  - Two-phase flows heat transfer and heat transfer coefficients predictions in evaporating and condensing flow
    - Homogeneous Equilibrium Model
    - Separated Flow Model
- This short course focuses on calculation methods for two phase pressure drop and heat transfer

TWO-PHASE SEPARATED FLOWS-BOILING

- Depiction of Convective Boiling Dominant Heat Transfer and Nucleate Boiling Dominant heat Transfer



One Dimensional Two Phase Flow

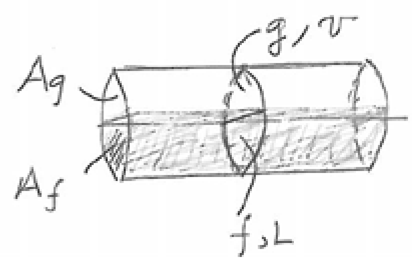
Definitions of Two-Phase Flow Parameters

- Area

One Dimensional Two-phase Flow

Area A_f, A_g [m²]

$$A = A_f + A_g$$



- Flow Rates

Mass flow rate

W_g, W_f [kg/s]

$$W = W_g + W_f$$

Mass Velocity

$$G = \frac{W}{A} \quad kg/m^2.s$$

Volumetric Flow rate.

Q_g, Q_f

$$Q = Q_g + Q_f$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

Phase Velocity

Phase velocity

$$u_g, u_f \quad [m/s]$$

$$u_g = \frac{Q_g}{A_g} \; ; \; u_f = \frac{Q_f}{A_f} = \frac{W_f}{\rho_f \cdot A_f}.$$

$$= \frac{Q_g \rho_g}{\rho_g \cdot A_g} = \frac{W_g}{\rho_g \cdot A_g}$$

Void Fraction

Void fraction – Volume based

$$\alpha_v = \frac{V_g}{V_f + V_g} =$$

Void fraction – Area Based

$$\alpha = \frac{A_g}{A_g + A_f} = A_g/A$$

Volumetric flow fraction β.

$$\beta = \frac{Q_g}{Q} = \frac{A_g u_g/u_g}{A_g u_g/u_g + A_f u_f/u_g} = \frac{A_g}{A_g + A_f\left(\frac{1}{S}\right)} \qquad S \equiv u_g/u_f \equiv \text{slip ratio}$$

For most applications $S > 1$

For $S = 1 \rightarrow$ Homogenous flow $\Rightarrow \beta = \alpha|_{\text{Area-Based}}$

Flow Quality

Flow Quality

$$x = \frac{W_g}{W_g+W_f} = \frac{\rho_g A_g u_g / \rho_g u_g A_g}{\frac{\rho_g A_g u_g}{\rho_g A_g u_g} + \frac{\rho_f u_f A_f}{\rho_g A_g u_g}} = \frac{1}{1+\frac{\rho_f}{\rho_g}\frac{A_f}{A_g}\frac{u_f}{u_g}}$$

$$= \frac{1}{1+\frac{\rho_f}{\rho_g}\frac{A_f}{A_g}\frac{1}{S}} \qquad \frac{A_f}{A_g} = \frac{A-A_g}{A_g} = \frac{A}{A_g} - 1 = \frac{1}{\alpha} - 1 = \left(\frac{1-\alpha}{\alpha}\right)$$

$$\Rightarrow x = \frac{1}{1+\frac{\rho_f}{\rho_g}\cdot\left(\frac{1-\alpha}{\alpha}\right)\cdot\frac{1}{S}}$$

$$x = \frac{1}{1+\frac{\rho_f}{\rho_g}\frac{A_f}{A_g}\frac{u_f}{u_g}} \Rightarrow x + x\frac{\rho_f}{\rho_g}\cdot\frac{A_f}{A_g}\frac{u_f}{u_g} = 1$$

$$\frac{A_f}{A_g} = \frac{1-x}{x\frac{\rho_f}{\rho_g}\frac{u_f}{u_g}} = \frac{1}{\alpha} - 1 \Rightarrow \frac{1}{\alpha} = 1 + \frac{1-x}{x\frac{\rho_f}{\rho_g}\frac{u_f}{u_g}}$$

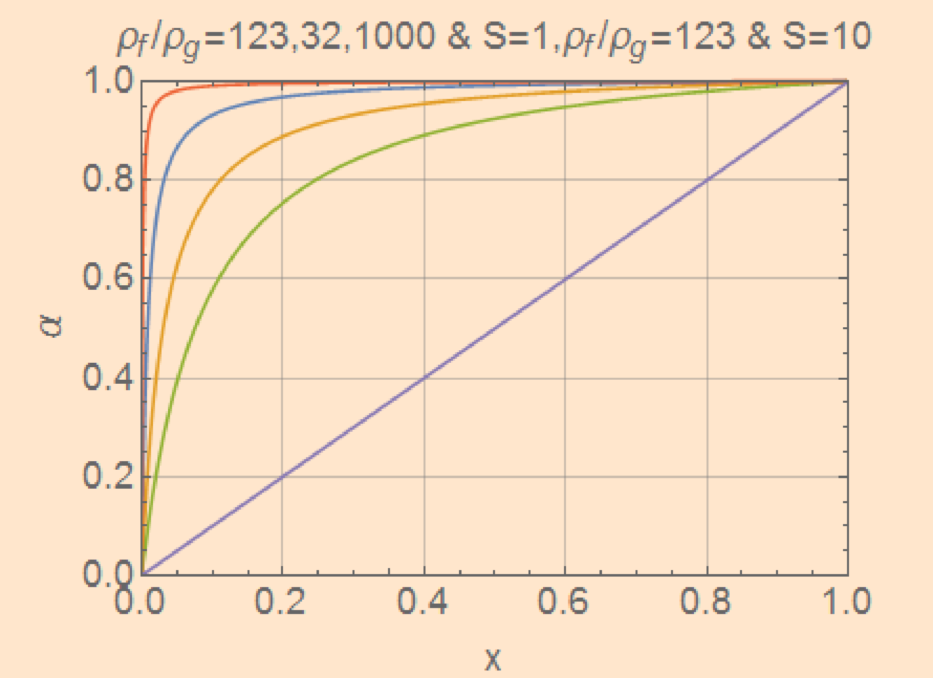
$$\Rightarrow \alpha = \frac{1}{1+\left(\frac{1-x}{x}\right)\frac{\rho_g}{\rho_f}S}$$

$\Rightarrow$ For a small $x \Rightarrow$ Large α

Calculation of Void Fraction from Flow Quality

```
α[x_, ρf_, ρg_, s_] := 1 / (1 + ((1-x)/x) (ρg/ρf) s)

Plot[{α[x, 1.6, .013, 1], α[x, 1.6, .05, 1], α[x, 1.6, .013, 10], α[x, 1, .001, 1], x}, {x, 0, 1}, Frame -> True, PlotRange → {{0, 1}, {0, 1}},
 FrameLabel → {"x", "α", "ρf/ρg=123,32,1000 & S=1,ρf/ρg=123 & S=10", ""}, LabelStyle → (FontSize → 18), AspectRatio → .7, GridLines → Automatic]
```



Density of Mixture

Mixture Density $\bar{\rho}$

$$\bar{\rho} = \frac{A_g}{A}\rho_g + \frac{A_f}{A}\rho_f = \alpha\rho_g + (1-\alpha)\rho_f \qquad \text{But } \alpha = \left(1 + \frac{\rho_g}{\rho_f}\left(\frac{1-x}{x}\right)\right)^{-1} \text{ for } S = 1$$

$$\frac{1}{\bar{\rho}} = \frac{1}{\alpha\rho_g + (1-\alpha)\rho_f} \qquad \alpha = \frac{1}{\rho_f x + \rho_g(1-x)} \cdot x \cdot \rho_f$$

$$\frac{1}{\bar{\rho}} = \frac{1}{\dfrac{x\rho_f \cdot \rho_g}{x\rho_f + \rho_g(1-x)} + \left(1 - \dfrac{x\rho_f}{x\rho_f + \rho_g(1-x)}\right)\cdot\rho_f}$$

$$= \frac{1}{\dfrac{x\rho_f\rho_g}{x\rho_f + \rho_g(1-x)} + \dfrac{\rho_f\left(x\rho_f + \rho_g(1-x) - x\rho_f\right)}{x\rho_f + \rho_g(1-x)}}$$

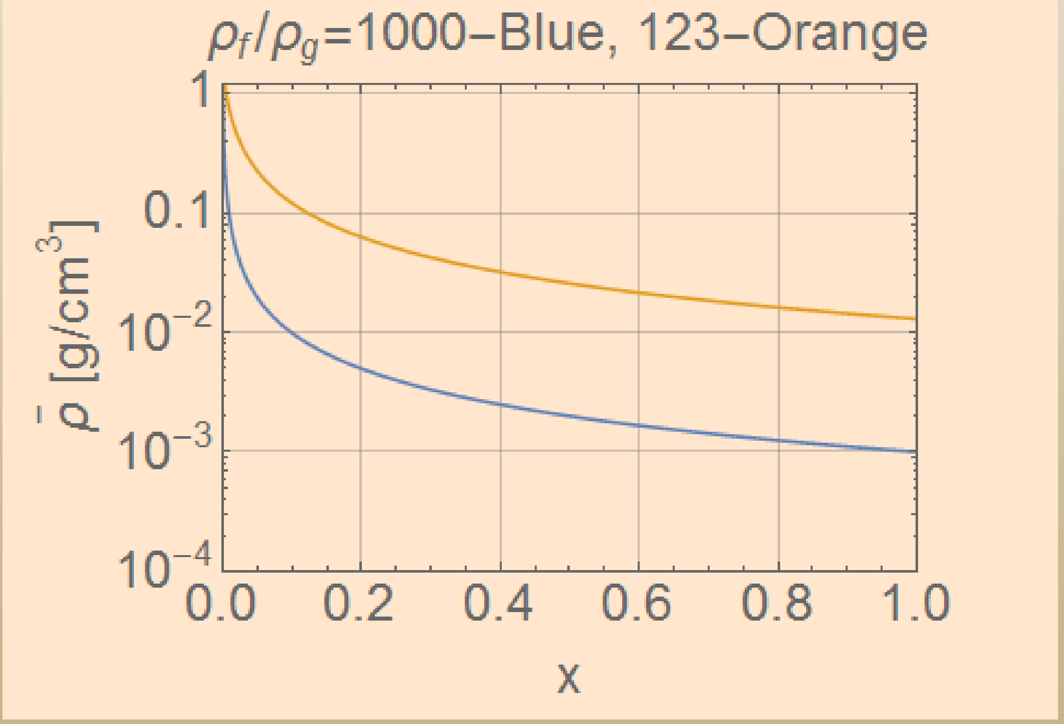
$$= \frac{x\rho_f + \rho_g(1-x)}{x\rho_f\rho_g + \cancel{x\rho_f^2} + \rho_f\rho_g(1-x) - \cancel{x\rho_f^2}}$$

$$= \frac{x\rho_f + \rho_g(1-x)}{x\rho_f\rho_g + \rho_f\rho_g(1-x)} = \frac{x\rho_f + \rho_g(1-x)}{\rho_f\rho_g} = \frac{x}{\rho_g} + \frac{1-x}{\rho_f}$$

$$= x v_g + (1-x)v_f = \bar{v}$$

+ Calculation of Average Density of Mixture

```
ρavg[x_, ρf_, ρg_] := 1 / (x / ρg + (1 - x) / ρf);
LogPlot[{ ρavg[x, 1., .001], ρavg[x, 1.6, .013]}, {x, 0, 1}, Frame -> True, PlotRange → {{0.0001, 1}, {0.0001, 1.2}},
 FrameLabel → {"x", "ρ̄ [g/cm^3]", "ρf/ρg=1000-Blue, 123-Orange", ""}, LabelStyle → (FontSize → 24), AspectRatio → .7, GridLines → Automatic]
```



Two Phase Flow Regime in a Heated Tube

Q_f = constant

q'' = constant

Flow quality

$$x = \frac{\rho_g u_g A_g}{\rho_g u_g A_g + \rho_f u_f A_f} = \frac{W_g}{W}$$

Thermodynamic equilibrium quality

"Mixing Cup" Quality

$$x_e = \frac{\bar{h} - h_f}{h_{fg}}$$

Mixture enthalpy

Homogeneous Two-Phase Equilibrium Model

For the Homogenous equilibrium flow

$S = 1 \Rightarrow x = x_e$ for $0 \leq x_e \leq 1$

$x \neq x_e$ because of the superheated Liquid Layer near the wall.

Homogeneous Two Phase Flow Model

Applicability to Bubble and Mist Flow

Assumptions.

Uniform Velocity $u_g = u_f = u \Rightarrow S = 1$

Uniform pressure $p_g = p_f = p$

Homogenous equilibrium Model

$T_g = T_f = T_{sat}|_{p=\text{local pressure}} \Rightarrow x_e = x \quad 0 \leq x_e \leq 1$

One Dimensional Conservation of Mass, Momentum and Energy

- Conservation of Mass

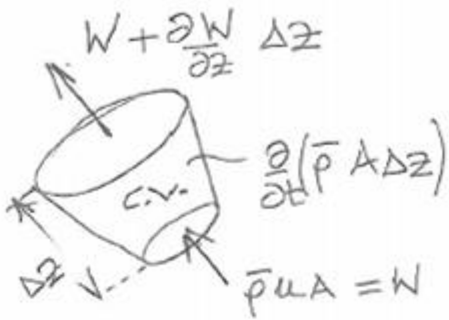
Conservation of Mass

Transport theorem

$$\frac{\partial}{\partial t}\int_v \bar{\rho}\, dv + \int_s \bar{\rho}\, \vec{u}_r \cdot d\vec{S} = 0$$

storage rate

Mass out − Mass in



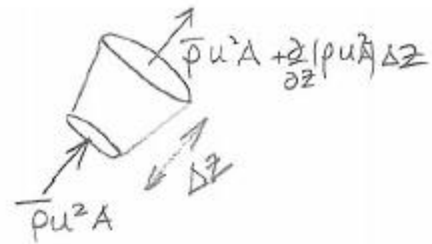
$$\frac{\partial}{\partial t}(\bar{\rho} A \Delta z) + \cancel{W} + \frac{\partial W}{\partial z}\Delta z - \cancel{W} = 0$$

$$\boxed{\frac{\partial}{\partial t}(\bar{\rho} A) + \frac{\partial W}{\partial z} = 0}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

+ Conservation of Momentum

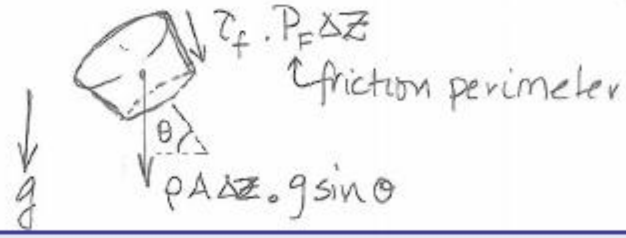
Momentum Along z direction. $\bar{\rho}u$



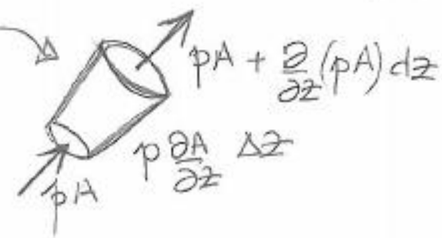
$$\frac{\partial}{\partial t}\int_V \bar\rho u \, dv + \int_S (\bar\rho u)\, u_r \cdot d\vec{s} = \sum_i F_z$$

storage — Mom. out − Mom. in = Net force.

Shear and Body forces



Pressure



$$\boxed{\frac{\partial}{\partial t}(\bar\rho u A) + \frac{\partial}{\partial z}(\rho u^2 A) = -A\frac{\partial p}{\partial z} - \tau_f P_f - \bar\rho g A \sin\theta}$$

+ Conservation of Energy

Conservation of energy.— $e^0 = e + \frac{u^2}{2} + g z \sin\theta$

$$\frac{J}{kg} = \frac{\cancel{kg}\,\frac{m}{s^2} \times m}{\cancel{kg}} \rightsquigarrow \frac{m^2}{s^2} \checkmark$$

Internal energy storage rate + Internal Energy out − In = Rate of Heat transferred to C.V. − Rate of work done by C.V.

$$\partial_t(\overline{\rho}\, h^0 A) + \partial_z(\overline{\rho}\, h^0 u A) = q'' P_H + q''' A + \partial_t(p A)$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

+ Assumptions

Steady State $\frac{\partial}{\partial t} = 0$; No chem. reaction $q''' = 0$

Tubular geometry $A = \text{const.}$

Neglect changes in kinetic and potential energy

Constant properties for individual phases

$$\frac{\partial}{\partial t}(\bar{\rho} A) + \frac{\partial W}{\partial z} = 0 \Rightarrow \boxed{\begin{array}{l} W = \text{const} = \bar{\rho} u A = G A \\ \text{Const. Area} \Rightarrow G = \text{constant} \end{array}}$$

Momentum

$$\cancelto{0}{\frac{\partial}{\partial t}(\bar{\rho} u A)} + \frac{\partial}{\partial z}(\bar{\rho} u^2 A) = -A \frac{\partial p}{\partial z} - \tau_F P_F - \bar{\rho} g A \sin\theta \; ; \; \bar{\rho} u A = G \cdot A = W$$

$$\boxed{W \frac{du}{dz} = -A \frac{dp}{dz} - \tau_F P_F - \bar{\rho} A g \sin\theta}$$

Energy

$$\cancelto{0}{\frac{\partial}{\partial t}(\bar{\rho} h^{o} A)} + \frac{\partial}{\partial z}(\bar{\rho} h^{o} u A) = q'' P_H + q''' A + \cancelto{0}{\frac{\partial}{\partial t}(p A)}$$

$$\boxed{\Rightarrow W \frac{dh}{dz} = P_H q''}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

+ Observations

1ϕ For Adiabatic Flow $u = \text{const.}$
" Flow w. Heat transfer $u = \text{const}$

2ϕ For Adiabatic Flow $u = \text{const.}$
" Flow w. Heat transfer Boiling → Flow accelerates
Condensation → Flow Decclerates

Remember $\bar{\rho} u = G \equiv \text{constant.}$
Boiling $x \nearrow, \alpha \nearrow, \bar{\rho} \searrow \Rightarrow u \uparrow$
Condensation $x \searrow, \alpha \searrow, \bar{\rho} \uparrow \Rightarrow u \searrow$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

+ Solution

$$\boxed{\text{Mass} + \text{Energy} \rightarrow x \rightarrow \text{Momentum} \rightarrow \Delta p}$$

$$\frac{dh}{dz} = \frac{q'' P_H}{W} \qquad h = h_f + x_e h_{fg}$$

$$h_{fg} \frac{dx_e}{dz} = \frac{q'' P_H}{W} \Rightarrow dx_e = \frac{q'' P_H}{W h_{fg}} dz$$

$$\Rightarrow x_e = x_{e,i} + \frac{P_H}{W h_{fg}} \int_0^z q'' d\xi$$

What is $x_{e,i}$ $\qquad x_{e,i}$ Thermodynamic Equilibrium quality at inlet

$$x_{e,i} = \frac{h_i - h_f}{h_{fg}} = -\frac{h_f - h_i}{h_{fg}} \qquad h_f - h_i = \Delta h_{sub,i}$$

$$= -\frac{c_{p,f}(T_{sat} - T_i)}{h_{fg}} = -\frac{c_{p,f}\,\Delta T_{sub,i}}{h_{fg}}$$

$$\Rightarrow \boxed{x_e = -\frac{c_{p,f}\,\Delta T_{sub}}{h_{fg}} + \frac{P_H}{W h_{fg}} \int_0^z q'' d\xi}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

+ Cases

$$h_i = h_f \Rightarrow x_{e,i} = 0$$

$$h_f < h_i < h_g \Rightarrow 0 < x_{e,i} < 1$$

$$h_i = h_g \Rightarrow x_{e,i} = 1$$

$$h_i > h_g \Rightarrow x_{e,i} > 1$$

For superheated region

$$x_{e,i} = \frac{h_i - h_f}{h_{fg}} = \frac{h_g - h_f + h_i - h_g}{h_{fg}} = \frac{h_{fg} + C_{p,g}(T_i - T_{sat})}{h_{fg}}$$

$$= 1 + \frac{C_{p,g}(T_i - T_{sat})}{h_{fg}}$$

$$x = \begin{cases} 0 & x_e < 0 \\ x_e & 0 \leq x_e \leq 1 \\ 1 & x_e > 1 \end{cases}$$

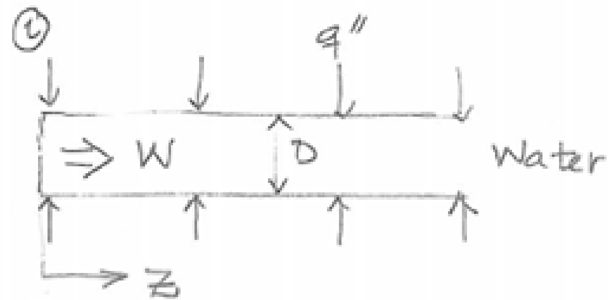
HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Uniformly heated Circular Tube
- Finding the z location where the thermodynamic quality $x_e = 0 \; and \; x_e = 1$

$T_i < 100°C.$

$$x_e(z) = -\frac{C_{p,f}\,\Delta T_{sub,i}}{h_{fg}} + \frac{P_H}{W h_{fg}}\int_0^z q''\,dz$$

$$= -\frac{C_{p,f}\,\Delta T_{sub}}{h_{fg}} + \frac{\pi D}{W h_{fg}}\,q''\,z$$



$T_i = 90°C$

$P_i = 1\,atm.$

$$\boxed{x_e = 0 \Rightarrow z\Big|_{x_e=0} = \frac{\dfrac{C_{p,f}\cdot\Delta T_{sub}}{\cancel{h_{fg}}}}{\dfrac{\pi D\, q''}{W\,\cancel{h_{fg}}}} = \frac{W\,C_{p,f}\,\Delta T_{sub}}{\pi D q''}}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Uniformly heated Circular Tube
- Finding x(z), α(z), u(z)

PURDUE UNIVERSITY

Homogeneous Two-Phase Flow Model – Steady State Solutions

Region	x	α	u
Subcooled $z < z\|_{x_e=0}$	0	0	$\dfrac{G}{\rho_f}$
Saturated $z\|_{x_e=0} < z < z\|_{x_e=1}$	x_e	$\dfrac{1}{1+\dfrac{\rho_g}{\rho_f}\left(\dfrac{1-x_e}{x_e}\right)}$	$G\left[x_e v_g + (1-x_e) v_f\right]$
Superheated $z > z\|_{x_e=1}$	1	1	$\dfrac{G}{\rho_g}$

October 2013 Short Course NASA Glenn Research Center | Course: Two-Phase Flow | Prof. Issam Mudawar

Example Problem Water Upward Flow in a Heated Pipe…

PURDUE UNIVERSITY

Numerical Example 1: Determination of Pressure Drop using HEM with Constant Two-Phase Friction Factor for Heated Vertical Upflow with Saturated Inlet

Saturated water ($x_e = 0$) at mass velocity $G = 250$ kg/m².s and inlet pressure of $p_i = 5$ bar enters a vertical circular tube of diameter $D = 1$ cm and length $L = 100$ cm, where it is subjected to a constant heat flux $q'' = 10^6$ W/m². Neglecting any kinetic or potential energy effects and assuming constant thermophysical properties, use the Homogeneous Equilibrium Model (HEM) with a constant two-phase friction factor $f_{TP} = 0.003$ to determine the following:

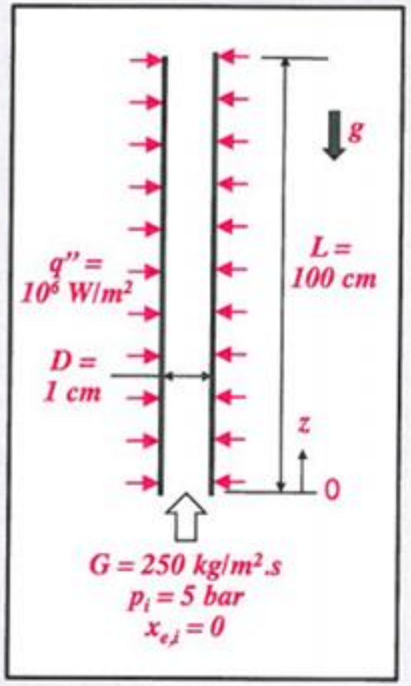
(a) $x_e(z), x_{e,L}$

(b) Δp_F

(c) Δp_A

(d) Δp_G

(e) Δp



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HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Finding $x_e[z]$ and the z location where the thermodynamic quality $x_e = 0$ and $x_e = 1$
- Finding $x_e[L]$
- Finding x[z] based on $x_e[z]$, and finding x'[z]

$$xe[z\_] := -\frac{Cpf\ \Delta Tsub}{hfg} + \frac{\pi\ DD\ q}{W\ hfg}\ z$$

$$zxe0 = \frac{W\ Cpf\ \Delta Tsub}{\pi\ DD\ q};$$

```
Print["z|xe=0 is ", zxe0]
```

z|x_e=0 is 0.

```
xe[1]; Print["xe[L]=", xe[L]]
```

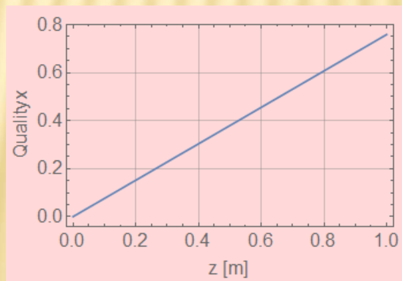
x_e[L]=0.759013

$$zxe1 = \frac{hfg\ W}{\pi\ DD\ q} + \frac{Cpf\ \Delta Tsub\ W}{\pi\ DD\ q};$$

```
If[zxe1 > L, zxe1 = L, zxe1];
```

```
p = 5 (*bar*);
Cpf = 4312 (*J/kg.K*);
hfg = 2.108 × 10^6 (*J/kg*);
vf = .0011 (*m^3/kg*);
vg = .3748 (*m^3/kg*);
μf = 180.1 × 10^-6 (*kg/m.s*); μg = 14.06 × 10^-6 (*kg/m.s*);
q = 1.0 × 10^6 (*W/m^2*); ΔTsub = 0 (*°C*);
g = 9.8 (*m.s^-2*);
θ = 90 / 180 π;
DD = .01 (*m*);
L = 1 (*m*);
G = 250 (*kg/m^2.s*);
W = G π (DD^2/4);
A = π DD^2/4 (*m^2*);
peri = π DD (*m*); DF = 4 A/peri (*m*);
vfg = vg - vf;
ReyNum = G DF / μf;
```

```
x[z_] := Piecewise[{{0, z < zxe0}, {xe[z], z > zxe0 && z < zxe1}, {Min[xe[L], 1], z > zxe1}}]
xp[z_] := x'[z]
Plot[x[z], {z, 0, L}, Frame → True, FrameLabel → {"z [m]", "Quality x"}, GridLines → Automatic, LabelStyle → (FontSize → 16)]
```



$$x = \begin{cases} 0 & x_e < 0 \\ x_e & 0 \le x_e \le 1 \\ 1 & x_e > 1 \end{cases}$$

Pressure Drop in Two Phase Homogeneous Equilibrium Model

$$W\frac{du}{dz} = -A\frac{dp}{dz} - \tau_F P_F - \bar{\rho} g A \sin\theta$$

From Conservation of Momentum

$$-\frac{dp}{dz} = \underbrace{\frac{\tau_F P_F}{A}}_{\substack{\text{Frictional} \\ -\left.\frac{dp}{dz}\right|_F}} + \underbrace{\frac{W}{A}\frac{du}{dz}}_{\substack{\text{Acceleration} \\ -\left.\frac{dp}{dz}\right|_A}} + \underbrace{\bar{\rho} g \sin\theta}_{\substack{\text{Gravitational.} \\ -\left.\frac{dp}{dz}\right|_G}}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Frictional Pressure Drop

$$-\frac{dp}{dz}\Big|_F = \frac{\tau_f P_F}{A} \qquad \text{Given} \quad D_H = \frac{4A}{P} \qquad P = \frac{4A}{D_H}$$

$$\Rightarrow -\frac{dp}{dz} = \frac{\tau_f 4\cancel{A}}{D_F \cancel{A}} = \frac{4}{D_F}\left(f_{TF}\frac{1}{2}\bar{\rho}u^2\right)$$

$$\Rightarrow \boxed{-\frac{dp}{dz} = \frac{2}{D_F} f_{TF} G^2 (v_f + x v_{fg})}$$

- Acceleration Pressure Drop

$$-\frac{dp}{dz}\Big|_A = W\frac{du}{dz} = \frac{W}{A}\frac{d}{dz}\left(\frac{W}{\bar{\rho}A}\right) = \frac{W^2}{A}\frac{d}{dz}\left(\frac{1}{\bar{\rho}}\right)$$

$$= \frac{W^2}{A}\frac{d}{dz}(v_f + x v_{fg}) = \boxed{\frac{W^2}{A} v_{fg}\frac{dx}{dz}}$$

- Pressure Drop due to Gravity

$$-\frac{dp}{dz}\Big|_G = \bar{\rho} g \sin\theta = \boxed{\frac{g\sin\theta}{(v_f + x v_{fg})}}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

Observations

- Pressure drop with vapor formation increases drastically
- For Adiabatic flows

  $$-\frac{dp}{dz}\Big|_A = 0 \quad \text{since} \quad \frac{dx}{dz} = 0$$

- Horizontal flow

  $$-\frac{dp}{dz}\Big|_G = 0$$

→ Adiabatic Horizontal flows are used to Determine the frictional gradient from measurements of total pressure gradient.

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

Two-phase Friction Factor

$$-\frac{dp}{dz}\Big|_F = \frac{2}{D_F} f_{TP}\, G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right)$$

By Analogy with single 1ϕ flow

$$f_{TP} = \frac{c}{\left(\frac{G D_F}{\bar{\mu}}\right)^n} = \underbrace{\frac{c}{\left(\frac{G D_F}{\mu_f}\right)^n}}_{f_{fo}\ \leftarrow \text{ Liquid only}} \left(\frac{\bar{\mu}}{\mu_f}\right)^n$$

$$f_{TP} = f_{fo} \cdot \left(\frac{\bar{\mu}}{\mu_f}\right)^n$$

Mixture viscosity

$$-\frac{dp}{dz}\Big|_F = \underbrace{\frac{2}{D_F} G^2 v_f f_{fo}}_{-\left(\frac{dp}{dz}\right)_{F_{fo}}} \cdot \frac{f_{TP}}{f_{fo}} \left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$-\left(\frac{dp}{dz}\right)_{F_{fo}} \left(\frac{\bar{\mu}}{\mu_o}\right)^n \left(1 + x\frac{v_{fg}}{v_f}\right)$$

$$-\left(\frac{dp}{dz}\right)_F = -\left(\frac{dp}{dz}\right)_{F_{fo}} \phi_{fo}^2 \leftarrow \text{Two phase friction Multiplier}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

- Pressure Drop Calculations/Constant Two-Phase Friction Factor

Use constant f_{TP}

$$-\frac{dp}{dz} = \frac{2}{D_F} f_{TP}\, G^2 v_f \left(1 + x\, v_g/v_f\right)$$

$$\boxed{.0029 < f_{TP} < .005}$$

- Pressure Drop Calculations/Using Two-Phase Viscosity Models

Using Viscosity Models

$$-\left(\frac{dp}{dz}\right)_F = \left\{\frac{2}{D_F} f_{fo}\, G^2 v_f\right\} \phi_{fo}^2$$

$$f_{fo} = \frac{c}{\left[\frac{G D_F}{\mu_f}\right]^n} \quad ; \quad \phi_{fo}^2 = \left(\frac{\bar{\mu}}{\mu_f}\right)^n \left(1 + x \frac{v_{fg}}{v_f}\right)$$

McAdams (1942) $\frac{1}{\bar{\mu}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f}$

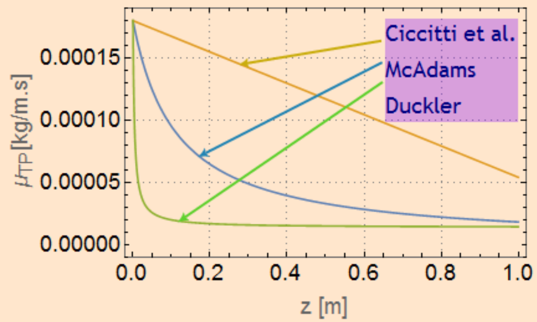
Ciccitti et al (1960) $\bar{\mu} = x\mu_g + (1-x)\mu_f$

Duckler (1964) $\bar{\mu} = \frac{x v_g \mu_g + (1-x) v_f \mu_f}{x v_g + (1-x) v_f}$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

Two-Phase Viscosity Models

```
μMA[z_] := μg μf / (x[z] μf + (1 - x[z]) μg) (*kg/m.s*); "McAdams";
μC[z_] := x[z] μg + (1 - x[z]) μf (*kg/m.s*); "Ciccitti et al.";
μD[z_] := (x[z] vg μg + (1 - x[z]) vf μf) / (x[z] vg + (1 - x[z]) vf) (*kg/m.s*); "Duckler";
Plot[{ μMA[z], μC[z], μD[z]}, {z, 0, 1}, Frame → True, FrameLabel → {"z [m]", "μTP[kg/m.s]", "", ""}, LabelStyle → (FontSize → 18),
 FrameTicks → Automatic, FrameTicksStyle → Black, GridLines → Automatic, GridLinesStyle → Directive[Dotted, Gray]]
```




```
coef = If[ReyNum < 2300, {c = 16, n = 1}, If[4 × 10^3 < ReyNum < 2 × 10^4, {c = .079, n = .25}],
  If[ReyNum > 2 × 10^4, {c = .046, n = .2}]]
```

Total Pressure Drop

$$-\left(\frac{dp}{dz}\right)_{Total} = -\left(\frac{dp}{dz}\right)_{F} + -\left(\frac{dp}{dz}\right)_{A} + -\left(\frac{dp}{dz}\right)_{G}$$

$$= \frac{2}{D_F} f_{TP} G^2 v_f \left(1 + x\frac{v_{fg}}{v_f}\right) + G^2 v_{fg}\frac{dx}{dz} + \frac{g\sin\theta}{v_f\left(1 + x\frac{v_{fg}}{v_f}\right)}$$

$$\Delta p(z) = \Delta p_{liquid\ phase} + \left[\int_{z|x_e=0}^{z} \left\{ \frac{2}{D_F} c\left(\frac{G D_F}{\mu_f}\right)^{-n} \left(\frac{\overline{\mu}(\xi)}{\mu_f}\right)^{n} G^2 v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right) + G^2 v_{fg}\left(\frac{dx(\xi)}{dz}\right) + \frac{g\sin(\theta)}{v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right)} \right\} d\xi \right] + \Delta p_{Vapor\ phase}$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

Example Problem Water Upward Flow in a Heated Pipe…

PURDUE UNIVERSITY

Numerical Example 1: Determination of Pressure Drop using HEM with Constant Two-Phase Friction Factor for Heated Vertical Upflow with Saturated Inlet

Saturated water ($x_e = 0$) at mass velocity $G = 250$ kg/m².s and inlet pressure of $p_i = 5$ bar enters a vertical circular tube of diameter $D = 1$ cm and length $L = 100$ cm, where it is subjected to a constant heat flux $q'' = 10^6$ W/m². Neglecting any kinetic or potential energy effects and assuming constant thermophysical properties, use the Homogeneous Equilibrium Model (HEM) with a constant two-phase friction factor $f_{TP} = 0.003$ to determine the following:

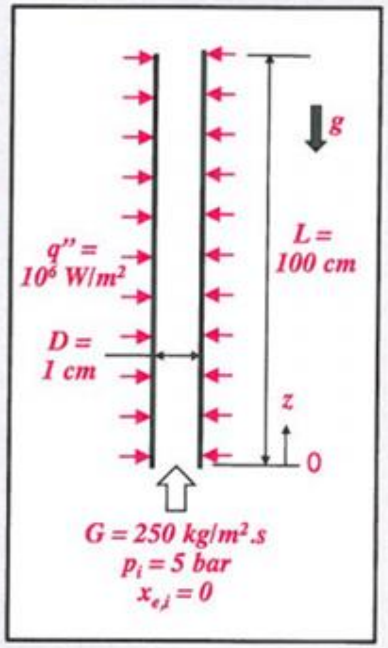
(a) $x_e(z)$, $x_{e,L}$

(b) Δp_F

(c) Δp_A

(d) Δp_G

(e) Δp



October 2013 Short Course NASA Glenn Research Center | Course: Two-Phase Flow | Prof. Issam Mudawar

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

Forming the Pressure Gradients Integrands

$$\text{intMA}[z\_] := \frac{2}{DF}\, c \left(\frac{G\,DF}{\mu f}\right)^{-n} \left(\frac{\mu MA[z]}{\mu f}\right)^{n} G^2\, vf \left(1+\frac{x[z]\, vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\, \text{Sin}[\theta]}{vf\, (1+x[z]\, vfg/vf)};$$

$$\text{intC}[z\_] := \frac{2}{DF}\, c \left(\frac{G\,DF}{\mu f}\right)^{-n} \left(\frac{\mu C[z]}{\mu f}\right)^{n} G^2\, vf \left(1+\frac{x[z]\, vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\, \text{Sin}[\theta]}{vf\, (1+x[z]\, vfg/vf)}$$

$$\text{intD}[z\_] := \frac{2}{DF}\, c \left(\frac{G\,DF}{\mu f}\right)^{-n} \left(\frac{\mu D[z]}{\mu f}\right)^{n} G^2\, vf \left(1+\frac{x[z]\, vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\, \text{Sin}[\theta]}{vf\, (1+x[z]\, vfg/vf)}$$

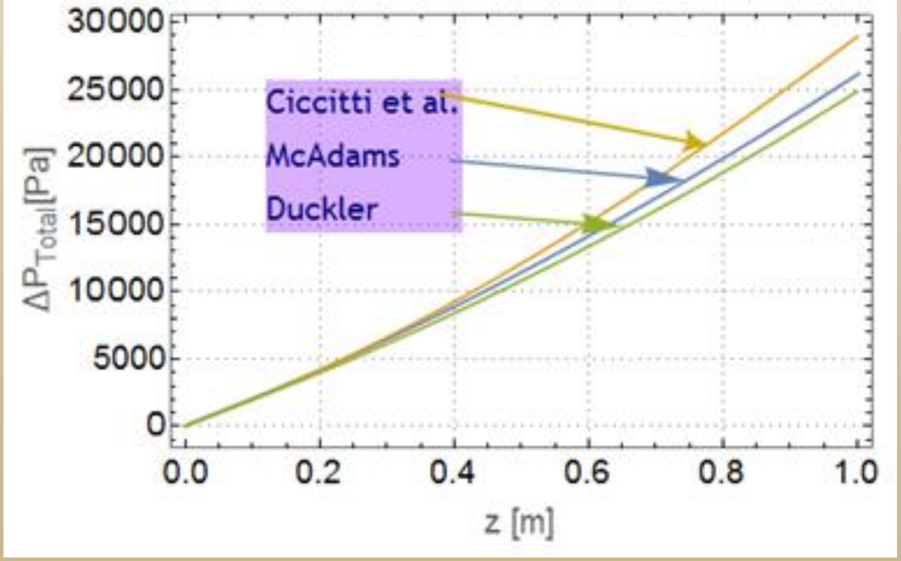
Numerical Integration

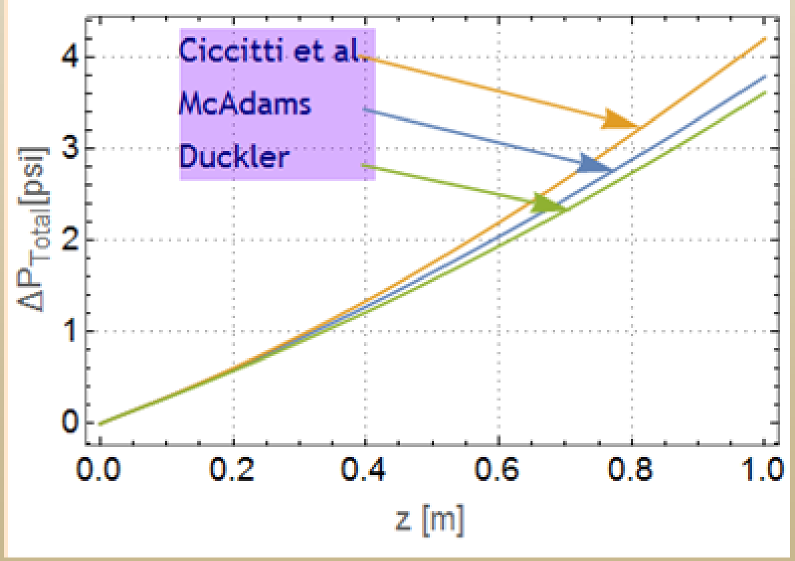
```
ΔPMA[zz_] := NIntegrate[intMA[z], {z, zxe0 + .00001, zz}];
ΔPC[zz_] := NIntegrate[intC[z], {z, zxe0 + .00001, zz}];
ΔPD[zz_] := NIntegrate[intD[z], {z, zxe0 + .00001, zz}];
```

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

Plotting ΔP as a Function of z

```
pPa = Plot[{ ΔPMA[z], ΔPC[z], ΔPD[z]}, {z, zxe0, zxe1}, Frame → True, FrameLabel → {"z  [m]", "ΔPTotal [Pa]", "", ""}, LabelStyle → (FontSize → 18),
  FrameTicks → Automatic, FrameTicksStyle → Black, GridLines → Automatic, GridLinesStyle → Directive[Dotted, Gray]]

ppsi = Plot[{ ΔPMA[z] / (1.013 × 10^5) 14.7, ΔPC[z] 1 / (1.013 × 10^5) × 14.7, ΔPD[z] / (1.013 × 10^5) 14.7}, {z, zxe0, zxe1}, Frame → True,
  FrameLabel → {"z  [m]", "ΔPTotal [psi]", "", ""}, LabelStyle → (FontSize → 18), FrameTicks → Automatic, FrameTicksStyle → Black, GridLines → Automatic,
  GridLinesStyle → Directive[Dotted, Gray]]
```





Cases Using the Homogenous Equilibrium Model

$$\Delta p(z) = \Delta p_{liquid\ phase} + \Bigg[\int_{z|x_e=0}^{z} \left\{ \frac{2}{D_F}\, c \left(\frac{G\, D_F}{\mu_f}\right)^{-n} \left(\frac{\overline{\mu}(\xi)}{\mu_f}\right)^{n} G^2\, v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right) + G^2\, v_{fg}\left(\frac{dx(\xi)}{dz}\right) + \frac{g\, \sin(\theta)}{v_f\left(1+x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right)} \right\} d\,\xi \Bigg] + \Delta p_{Vapor\ phase}$$

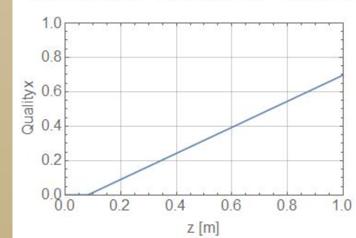
```
p = 5 (*bar*);
Cpf = 4312 (*J/kg.K*);
hfg = 2.108 × 10^6 (*J/kg*);
vf = .0011 (*m^3/kg*);
vg = .3748 (*m^3/kg*);
μf = 180.1 × 10^-6 (*kg/m.s*);
μg = 14.06 × 10^-6 (*kg/m.s*);
q = 1.0 × 10^6 (*W/m^2*);
ΔTsub = 30 (*°C*);
g = 9.8 (*m.s^-2*);
θ = 90 / 180 π;
DD = .01 (*m*);
L = 1 (*m*);
G = 250 (*kg/m^2.s*);
W = G π (DD^2/4);
A = π DD^2/4
(*m^2*);
```

```
peri = π DD (*m*); DF = 4 A/peri (*m*);
vfg = vg - vf; ReyNum = G DF / μf;
ReyNumg = G DF / μg;
```

Finding $x_e[z]$ and the z location where the thermodynamic quality x_e=0 and x_e=1

```
xe[z_] := - Cpf ΔTsub/hfg + π DD q/(W hfg) z

zxe0 = W Cpf ΔTsub/(π DD q); L1ph = zxe0;

xe[L]

0.697647

zxe1 = hfg W/(π DD q) + Cpf ΔTsub W/(π DD q); L2ph = zxe1;
If[L2ph < L, intL = L2ph, intL = L];
```

```
x[z_] :=
  Piecewise[{{0, z < zxe0}, {xe[z], z > zxe0 && z < zxe1},
    {Min[xe[L], 1], z > zxe1}}]

x[z_] :=
  Piecewise[{{0, z < L1ph}, {xe[z], z > L1ph && z < intL},
    {Min[xe[L], 1], z > intL}}]

xp[z_] := x'[z]

Plot[x[z], {z, 0, L + .5 L}, Frame → True,
 FrameLabel → {"z [m]", "Quality x"}, GridLines → Automatic,
 LabelStyle → (FontSize → 16), PlotRange → {{0, L}, {0, 1}}]
```



HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

$$\mu MA[z\_] := \frac{\mu g\,\mu f}{x[z]\,\mu f + (1 - x[z])\,\mu g} \;(*kg/m.s*);\; \text{"McAdams"};$$

$$\mu C[z\_] := x[z]\,\mu g + (1 - x[z])\,\mu f \;(*kg/m.s*);$$

"Ciccitti et al.";

$$\mu D[z\_] := \frac{x[z]\,vg\,\mu g + (1 - x[z])\,vf\,\mu f}{x[z]\,vg + (1 - x[z])\,vf} \;(*kg/m.s*);\; \text{"Duckler"};$$

```
LogPlot[{ μMA[z], μC[z], μD[z]}, {z, 0, 1}, Frame → True,
 FrameLabel → {"z [m]", "μTP[kg/m.s]", "", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
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 GridLinesStyle → Directive[Dotted, Gray]]
```
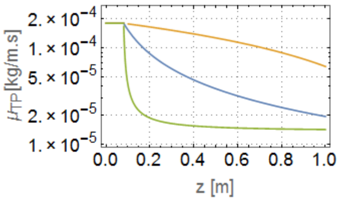




```
If[ReyNum < 2300, {c = 16, n = 1}]
If[2.3 × 10^3 < ReyNum < 2 × 10^4, {c = .079, n = .25}]
If[ReyNum > 2 × 10^4, {c = .046, n = .2}]
{0.079, 0.25}

If[ReyNumg < 2300, {c = 16, n = 1}]
If[4 × 10^3 < ReyNumg < 2 × 10^4, {c = .079, n = .25}]
If[ReyNumg > 2 × 10^4, {c = .046, n = .2}]
{0.046, 0.2}
```

$$\Delta p(z) = \Delta p_{liquid\ phase} + \left[\int_{z|x_e=0}^{z} \left\{ \frac{2}{D_F}\, c\left(\frac{G\,D_F}{\mu_f}\right)^{-n} \left(\frac{\overline{\mu}(\xi)}{\mu_f}\right)^{n} G^2\, v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right) + G^2\, v_{fg}\left(\frac{dx(\xi)}{dz}\right) + \frac{g\,\sin(\theta)}{v_f\left(1 + x(\xi)\left(\frac{v_{fg}}{v_f}\right)\right)} \right\} d\,\xi \right] + \Delta p_{Vapor\ phase}$$

$$intMA[z\_] := \frac{2}{DF}\, c \left(\frac{G\,DF}{\mu f}\right)^{-n} \left(\frac{\mu MA[z]}{\mu f}\right)^{n} G^2\, vf \left(1 + \frac{x[z]\,vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\,Sin[\theta]}{vf\,(1 + x[z]\,vfg / vf)};$$

$$intC[z\_] := \frac{2}{DF}\, c \left(\frac{G\,DF}{\mu f}\right)^{-n} \left(\frac{\mu C[z]}{\mu f}\right)^{n} G^2\, vf \left(1 + \frac{x[z]\,vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\,Sin[\theta]}{vf\,(1 + x[z]\,vfg / vf)}$$

$$intD[z\_] := \frac{2}{DF}\, c \left(\frac{G\,DF}{\mu f}\right)^{-n} \left(\frac{\mu D[z]}{\mu f}\right)^{n} G^2\, vf \left(1 + \frac{x[z]\,vfg}{vf}\right) + G^2\, vfg\, xp[z] + \frac{g\,Sin[\theta]}{vf\,(1 + x[z]\,vfg / vf)}$$

$$\Delta PMA[zz\_] := NIntegrate[intMA[z], \{z, zxe0 + .00001, zz\}] + \frac{2\,c\,ReyNum^{-n}\,vf\,G^2\,L1ph}{DF} + \frac{2\,c\,ReyNumg^{-n}\,vf\,G^2\,(L - intL)}{DF};$$

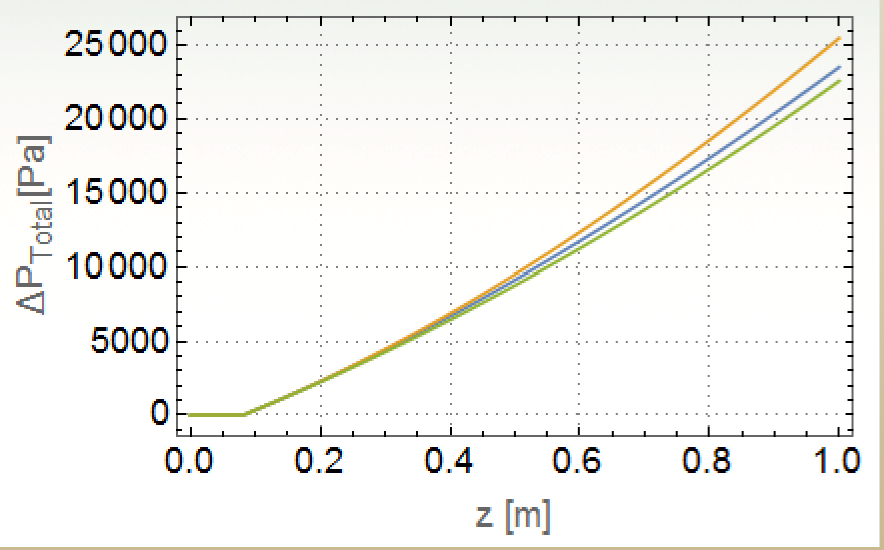
$$\Delta PC[zz\_] := NIntegrate[intC[z], \{z, zxe0 + .00001, zz\}] + \frac{2\,c\,ReyNum^{-n}\,vf\,G^2\,L1ph}{DF} + \frac{2\,c\,ReyNumg^{-n}\,vf\,G^2\,(L - intL)}{DF};$$

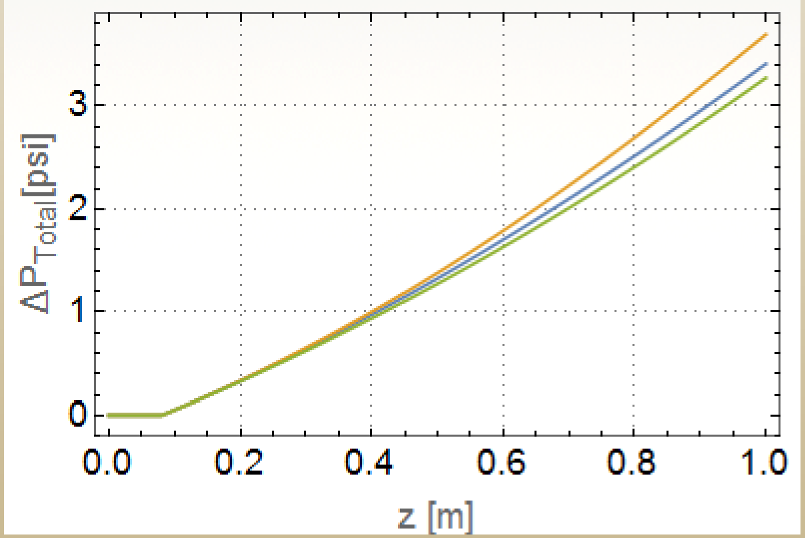
$$\Delta PD[zz\_] := NIntegrate[intD[z], \{z, zxe0 + .00001, zz\}] + \frac{2\,c\,ReyNum^{-n}\,vf\,G^2\,L1ph}{DF} + \frac{2\,c\,ReyNumg^{-n}\,vf\,G^2\,(L - intL)}{DF};$$

HYDRODYNAMICS AND PRESSURE DROP IN TWO-PHASE EVAPORATING AND CONDENSING FLOWS

```
Plot[{ΔPMA[z], ΔPC[z], ΔPD[z]}, {z, 0, L}, Frame → True,
 FrameLabel → {"z [m]", "ΔPTotal [Pa]", "", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```

```
Plot[{ΔPMA[z] / (1.013×10^5) 14.7, ΔPC[z] 1/ (1.013×10^5) ×14.7,
  ΔPD[z] / (1.013×10^5) 14.7}, {z, 0, L}, Frame → True,
 FrameLabel → {"z [m]", "ΔPTotal [psi]", "", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```



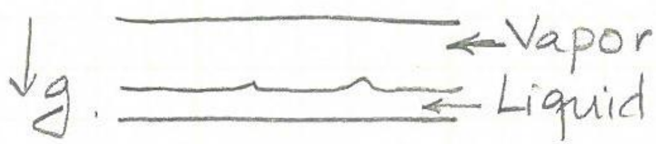


- Pressure Drop in Separated Flows-Slip Flow Model

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Features
  - Allows for differences in phase velocities
  - Intended for annular and stratified flows.
  - Separate Analyses of individual phases
- Assumptions
  - Different but Uniform phase velocities

$$u_g \neq u_f \qquad S \neq 1$$



  - Uniform pressure over entire flow area

$$p_g = p_f = p$$

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

Basic Relations

$$u_g = \frac{Q_g}{A_g} \times \frac{\rho_g}{\rho_g} = \frac{W_g}{\rho_g A_g}$$

but $A_g/A = \alpha \Rightarrow A_g = \alpha A$

$$\frac{W_g}{W} = x \Rightarrow W_g = x W$$

$$\frac{W}{A} = G$$

$$\Rightarrow u_g = \frac{W_g}{\rho_g \alpha A} = \frac{x W}{\rho_g \alpha A}$$

$$u_g = \frac{x G}{\alpha \rho_g}$$

$$u_f = \frac{(1-x) G}{(1-\alpha)\rho_f}$$ from $u_f = \frac{Q_f}{A_f}$

$\bar{\rho} = \alpha \rho_g + (1-\alpha)\rho_f$ However $S \neq 1$

In the Homogenous equilibrium model, we derived

$$\frac{1}{\bar{\rho}} = \bar{v} = x v_g + (1-x) v_f$$ ← This was based on $S = 1$

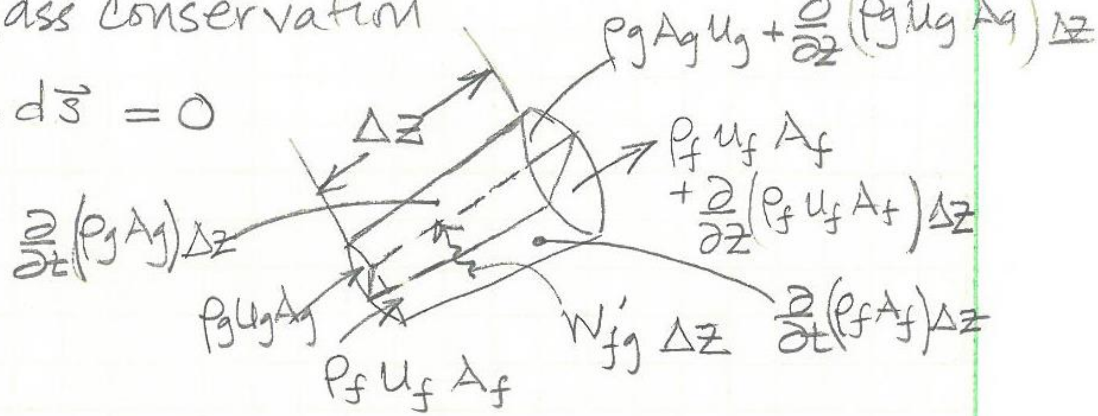
$$\Rightarrow \frac{1}{\bar{\rho}} \neq \bar{v}$$ because $S \neq 1$

Void fraction becomes an unknown in this formulation

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Conservation Laws / Mass Conservation
  - Transport Theorem

$$\frac{\partial}{\partial t}\int_V \rho \, dv + \int_S \rho \, \vec{u}_r \cdot d\vec{S} = 0$$



- Vapor phase

$$\frac{\partial}{\partial t}(\rho_g A_g) + \frac{\partial}{\partial z}(\rho_g u_g A_g) - W'_{fg} = 0$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho_g \alpha A) + \frac{\partial}{\partial z}(x W) - W'_{fg} = 0$$

- Liquid phase

$$\frac{\partial}{\partial t}(\rho_f A_f) + \frac{\partial}{\partial z}(\rho_f A_f u_f) + W'_{fg} = 0$$

$$\Rightarrow \frac{\partial}{\partial t}\left[\rho_f (1-\alpha) A\right] + \frac{\partial}{\partial z}\left[(1-x) W\right] + W'_{fg} = 0$$

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Combining the two phases

$$\frac{\partial}{\partial t}\Big[\underbrace{\rho_g \alpha A + \rho_f (1-\alpha) A}_{\bar{\rho} A}\Big] + \frac{\partial}{\partial z}(W) = 0$$

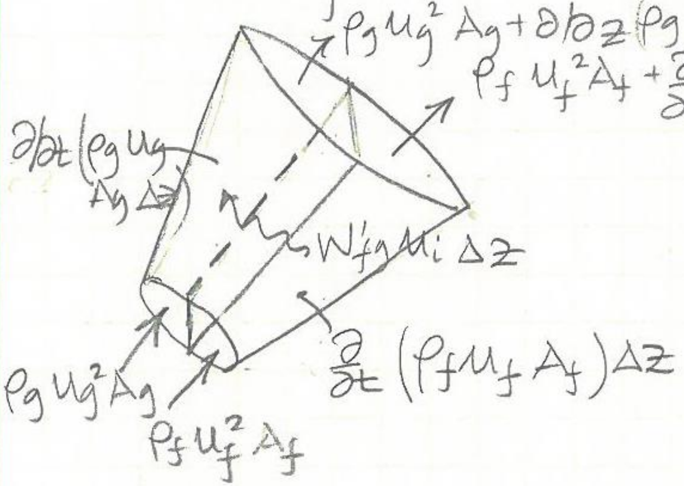
$$\Rightarrow \frac{\partial}{\partial t}\left[\bar{\rho} A\right] + \frac{\partial}{\partial z}(W) = 0$$

W'_{fg} cancelled when combining both phases

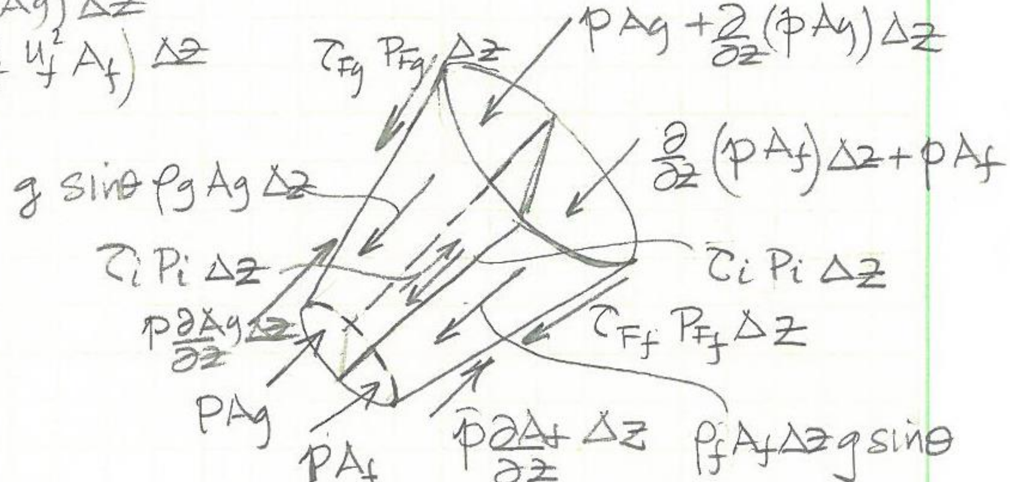
TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

1) Momentum Conservation

$$\underbrace{\partial/\partial t \int_V (\rho u)\, dv}_{\text{Rate of Mom. storage}} + \underbrace{\int_S (\rho u)\, \vec{u}_r \cdot d\vec{s}}_{\text{Mom. Out} - \text{Mom. in}} = \underbrace{\sum_i F_z}_{\text{Net force on C.V.}}$$



Conservation of Momentum



Forces

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Vapor phase

$$\partial_t(\rho_g u_g \alpha A) + \partial_z(\rho_g u_g^2 \alpha A) - W'_{fg} u_i = -\alpha A \partial_z p - \tau_{F_g} P_{F_g} - \tau_i P_i - \rho_g \alpha A g \sin\theta$$

- Liquid phase

$$\partial_t(\rho_f u_f (1-\alpha) A) + \partial_z(\rho_f u_f^2 (1-\alpha) A) + W'_{fg} u_i$$
$$= -(1-\alpha) A \partial_z p - \tau_{F_f} P_{F_f} + \tau_i P_i - \rho_f (1-\alpha) A g \sin\theta$$

- Combined

$$\partial_t(W) + \partial_z\left\{\left[\rho_g \alpha u_g^2 + \rho_f (1-\alpha) u_f^2\right] A\right\}$$
$$= -A \partial_z p - \tau_{F_g} P_{F_g} - \tau_{F_f} P_{F_f} - \left[\rho_g \alpha + \rho_f (1-\alpha)\right] A g \sin\theta$$

Interfacial terms cancel out –

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Mass, Energy & Momentum
  - Mass $\partial_t(\bar{\rho} A) + \partial_z(W) = 0$
  - Momentum $\partial_t(W) + \partial_z\left(\left[\rho_g \alpha \left(\frac{xW}{\rho_g \alpha A}\right)^2 + \rho_f(1-\alpha)\left(\frac{(1-x)W}{\rho_f(1-\alpha)A}\right)^2\right]A\right)$

$$= -\partial_z p - \tau_{F_g} P_{F_g} - \tau_{F_f} \cdot P_{F_f} - \bar{\rho} A g \sin\theta$$

- Conservation of Energy.

Internal energy, heat, and work.

$$\partial_t\left[\rho_g h_g^o \alpha A + \rho_f h_f^o (1-\alpha) A\right] + \partial_z\left[x W h_g^o + (1-x) W h_f^o\right] = \left(q_g'' P_{H_g} + q_f'' P_{H_f}\right)$$

$$+ \left[q_g''' \alpha A + q_f'''(1-\alpha) A\right] + \partial_t(pA)$$

Here

$$h_g^o = h_g + u_g^2/2 + gz\sin\theta \qquad h_f^o = h_f + \frac{u_f^2}{2} + gz\sin\theta$$

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

○ Conservation of Energy.

Internal energy, heat, and work.

$$\partial_t\left[\rho_g h_g^o \alpha A + \rho_f h_f^o (1-\alpha) A\right] + \partial_z\left[x W h_g^o + (1-x) W h_f^o\right] = \left(q_g'' P_{Hg} + q_f'' P_{Hf}\right) + \left[q_g''' \alpha A + q_f''' (1-\alpha) A\right] + \partial_t (pA)$$

Here

$$h_g^o = h_g + u_g^2/2 + gz\sin\theta \qquad h_f^o = h_f + \frac{u_f^2}{2} + gz\sin\theta$$

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Steady State and other simplifications.

Steady state $\partial/\partial t\,(\ ) = 0 \Rightarrow$ Continuity yields

$\Rightarrow \partial_z W = 0 \quad \Rightarrow W = \text{const} = GA$. with A const

$\Rightarrow G = \text{constant}$

Neglecting kinetic and potential energy

$h_k^{o} \rightarrow h_k \qquad k = f, g \qquad x = x_e$

Momentum

$$G^2 \frac{d}{dz}\left(\frac{x^2}{\rho_g \alpha} + \frac{(1-x)^2}{\rho_f (1-\alpha)} \right) = -\frac{dp}{dz} - \frac{\tau_{F_g} P_{F_g}}{A} - \frac{\tau_{F_f} P_{F_f}}{A} - \bar{\rho} g \sin\theta$$

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

Energy

$$W \frac{dh}{dz} = q'' P_H \qquad h = h_f + x_e h_{fg}$$

$$q'' P_H = q''_g P_{Hg} + q''_f P_{Hf}$$

$$\Rightarrow \frac{dx_e}{dz} = \frac{q'' P_H}{W h_{fg}}$$

$$\Rightarrow x_e(z) = x_{e,i} + \frac{P_H}{W h_{fg}} \int_0^z q'' d\xi$$

Energy yielded quality as a function of z

We know $x[z]$

$\Rightarrow$ In the momentum equation, α and $\tau_F P_F$ are the unknowns

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

As in the previous formulation,

$$-\frac{dp}{dz} = -\frac{dp}{dz}\Big|_F + -\frac{dp}{dz}\Big|_{Acc} + -\frac{dp}{dz}\Big|_G$$

- Frictional

$$-\frac{dp}{dz}\Big|_F = \frac{\tau_F P_F}{A} = \left(\frac{2}{D_F} f_{fo}\, v_f\, G^2\right) \phi_{fo}^2$$

Liquid based only.

$$-\frac{dp}{dz}\Big]_G = \bar{\rho} g \sin\theta = \left(\alpha \rho_g + (1-\alpha)\rho_f\right) g \sin\theta$$

$$-\frac{dp}{dz}\Big)_A = G^2 \frac{d}{dz}\left(\frac{x^2 v_g}{\alpha} + \frac{(1-x)\, v_f}{(1-\alpha)}\right)$$

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Gravity

$$-\frac{dp}{dz}\bigg|_G = \bar{\rho} g \sin\theta = \left[\alpha \rho_g + (1-\alpha)\rho_f\right] g \sin\theta$$

- Acceleration

$$-\frac{dp}{dz} = G^2 \left[\frac{d}{dz}\left(\frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha}\right)\right]$$

When flashing is negligible

$$\Rightarrow \quad x \neq f(p)$$

TWO-PHASE SEPARATED FLOWS-SLIP FLOW MODEL

- Martinelli – Nelson Methods for Separated flow pressure drop

$$\Delta P = \sum_i \int \left(-\frac{dp}{dz}\right)_i dz = \Delta P_F + \Delta P_A + \Delta P_G.$$

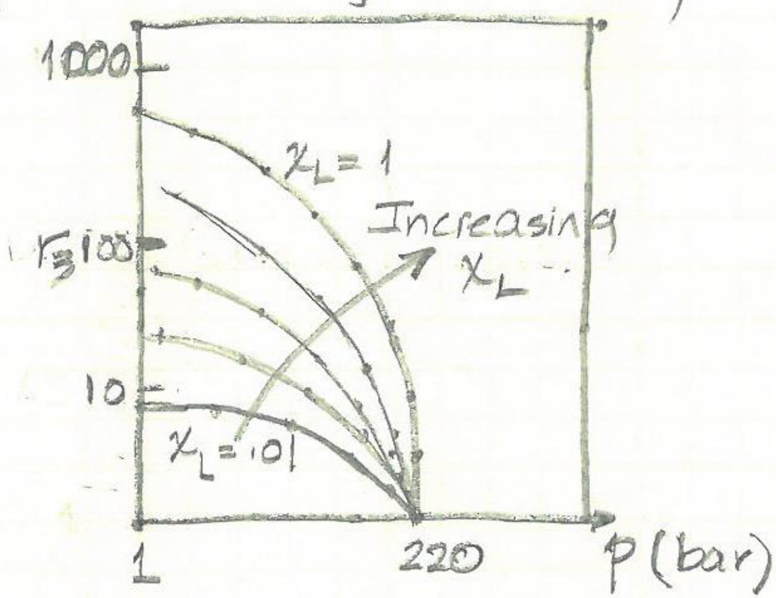
$$\Delta P_F = \left(\frac{2L}{D_F} f_{fo}\, G^2 v_f\right) r_3$$

$$\Delta P_A = \left(G^2 v_f\right) r_2$$

$$\Delta P_G = \left(\frac{g L \sin\theta}{v_f}\right) r_4$$

where

$r_2, r_3, r_4 = f(P_{in}, x_{e,L})$



PRESSURE DROP IN SEPARATED FLOWS

Pressure Drop in Separated Flows-Lockhart-Martinelli's Approach for Adiabatic Flows

PRESSURE DROP IN SEPARATED FLOWS

Separated Flow Pressure Drop Calculations

Lockhart – Martinelli Method.

- Assumptions
  - Low pressure
  - Horizontal
  - Adidbatic (air-water)
  - $-\frac{dp}{dz}\Big]_F$ only

Basis for development of new Correlations by many authors

$$-\frac{dp}{dz}\Big]_F = -\frac{dp}{dz}\Big|_f \times \phi_f^2 = -\frac{dp}{dz}\Big|_g \phi_g^2$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2 f_f G^2 (1-x)^2 v_f}{D_F} \; ; \; f_f = \frac{A}{Re_f^n} \; ; \; Re_f = \frac{G(1-x) D_F}{\mu_f}$$

$$-\left(\frac{dp}{dz}\right)_g = \frac{2 f_g G^2 x^2 v_g}{D_F} \; ; \; f_g = \frac{A}{Re_g^n} \; ; \; Re_g = \frac{G x D_F}{\mu_g}$$

PRESSURE DROP IN SEPARATED FLOWS

- Lockhart-Martinelli Parameter

$$X^2 = \frac{-\left.\frac{dp}{dz}\right|_f}{-\left.\frac{dp}{dz}\right|_g}$$

: Laminar $A = 16$ $n = 1$

Turbulent $A = .046$ $n = .2$

- Sequence of Calculation

– Given p, x, G, D_F

- Calculate $-\left.\frac{dp}{dz}\right|_f$; $\left.\frac{dp}{dz}\right|_g$
- Calculate X
- Determine C from table

Flow state Liquid-gas	C
Turbulent - Turbulent	20
Laminar - Turbulent	12
Turbulent - Laminar	10
Laminar - Laminar	5

- $\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$

- $-\left.\frac{dp}{dz}\right|_F = -\left.\frac{dp}{dz}\right|_f \cdot \phi_f^2$
- $\Delta p_F = \int_0^{L_{TP}} \left(-\frac{dp}{dz}\right)_F dz$

PRESSURE DROP IN SEPARATED FLOWS

Example Problems

"Fluid is FC-72"

$p = 2$ (*bar*);

$Cpf = 1136$ (*J/kg.K*);

$hfg = 87272$ (*J/kg*);

$vf = .0006515$ (*m^3/kg*);

$vg = .0387$ (*m^3/kg*);

$\mu f = 349.0 \times 10^{-6}$ (*kg/m.s*);

$\mu g = 12.3 \times 10^{-6}$ (*kg/m.s*);

$\sigma = .0062$ (*N/m*);

$q = 4.0 \times 10^{4}$ (*W/m^2*);

$\Delta Tsub = 0$ (*°C*);

$g = 9.8$ (*$m.s^{-2}$*);

$\theta = 0 / 180\, \pi$;

$DD = .005$ (*m*);

$L = .25$ (*m*);

$G = 250$ (*$kg/m^2.s$*);

$W = G\, \pi \left(\frac{DD^2}{4}\right)$;

$A = \frac{\pi\, DD^2}{4}$ (*m^2*);

$peri = \pi\, DD$ (*m*);

$DF = \frac{4\, A}{peri}$ (*m*);

$vfg = vg - vf$;

$ReyNum = G\, DF / \mu f$;

$ReyNumg = G\, DF / \mu g$;

Quality as a Function of z, $x_e(z)$

$xe[z\_] := -\frac{Cpf\, \Delta Tsub}{hfg} + \frac{\pi\, DD\, q}{W\, hfg}\, z$

$xe0 = xe[0]$

$xel = xe[L]$

0.

0.36667

$zxe0 = \frac{W\, Cpf\, \Delta Tsub}{\pi\, DD\, q}$; $L1ph = zxe0$;

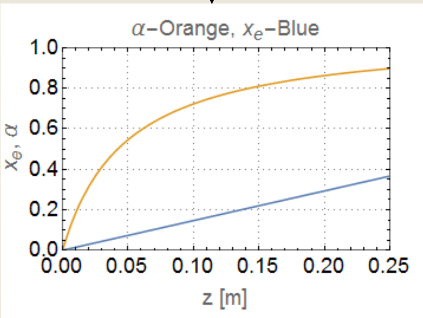
$zxe1 = \frac{hfg\, W}{\pi\, DD\, q} + \frac{Cpf\, \Delta Tsub\, W}{\pi\, DD\, q}$; $L2ph = zxe1$;

If[$L2ph < L$, $intL = L2ph$, $intL = L$];

Void Fraction and Quality (Zivi, 1964)

$$\alpha[z\_] := \left(1 + \frac{1 - xe[z]}{xe[z]} \left(\frac{vf}{vg}\right)^{2/3}\right)^{-1}$$

```
Plot[{xe[z], α[z]}, {z, zxe0 + .00001, intL},
  Frame → True,
  FrameLabel → {"z [m]", "xe, α",
     "α-Orange, xe-Blue", ""},
  LabelStyle → (FontSize → 18),
  FrameTicks → Automatic, FrameTicksStyle → Black,
  GridLines → Automatic,
  GridLinesStyle → Directive[Dotted, Gray],
  PlotRange → {{0, L}, {0, 1}}]
```



Friction Factors on the liquid and gas sides

$ff[z\_] :=$

$$\text{Piecewise}\Big[\Big\{\Big\{16 \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right)^{-1}, \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right) < 2000\Big\}, \Big\{.079 \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right)^{-.25}, 2000 < \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right) < 20\,000\Big\}, \Big\{.046 \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right)^{-.2}, \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right) > 20\,000\Big\}\Big\}\Big]$$

$fg[z\_] :=$

$$\text{Piecewise}\Big[\Big\{\Big\{16 \left(\frac{G\,(xe[z])\, DD}{\mu g}\right)^{-1}, \left(\frac{G\,(xe[z])\, DD}{\mu g}\right) < 2000\Big\}, \Big\{.079 \left(\frac{G\,(xe[z])\, DD}{\mu g}\right)^{-.25}, 2000 < \left(\frac{G\,(xe[z])\, DD}{\mu g}\right) < 20\,000\Big\}, \Big\{.046 \left(\frac{G\,(xe[z])\, DD}{\mu g}\right)^{-.2}, \left(\frac{G\,(xe[z])\, DD}{\mu g}\right) > 20\,000\Big\}\Big\}\Big]$$

Constant C

$CC[z\_] :=$

$$\text{Piecewise}\Big[\Big\{\Big\{5, \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right) < 2000\ \&\&\ \left(\frac{G\,(xe[z])\, DD}{\mu g}\right) < 2000\Big\}, \Big\{12, \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right) < 2000\ \&\&\ 2000 < \left(\frac{G\,(xe[z])\, DD}{\mu g}\right)\Big\}, \Big\{10, 2000 < \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right)\ \&\&\ \left(\frac{G\,(xe[z])\, DD}{\mu g}\right) < 2000\Big\}, \Big\{20, 2000 < \left(\frac{G\,(1 - xe[z])\, DD}{\mu f}\right)\ \&\&\ 2000 < \left(\frac{G\,(xe[z])\, DD}{\mu g}\right)\Big\}\Big\}\Big]$$

PRESSURE DROP IN SEPARATED FLOWS

Frictional, Acceleration and Gravitational Pressure Gradients

$$\text{dpdzF}[z\_] := \frac{1}{\text{DD}} 2\, G^2\, \text{vf ff}[z]\, (1-\text{xe}[z])^2 \left(1 + \frac{\text{CC}[z]}{\sqrt{\frac{\text{vf ff}[z]\,(1-\text{xe}[z])^2}{\text{vg fg}[z]\,\text{xe}[z]^2}}} + \frac{\text{vg fg}[z]\,\text{xe}[z]^2}{\text{vf ff}[z]\,(1-\text{xe}[z])^2}\right)$$

$$\text{dpdzA}[z\_] := G^2\, \text{vf} \left(\frac{(\text{xe}[z])^2\, \text{vg}}{\alpha[z]\, \text{vf}} + \frac{(1-(\text{xe}[z]))^2}{(1-\alpha[z])} - 1\right)$$

$$\text{dpdzG}[z\_] := \left(\frac{\alpha[z]}{\text{vg}} + \frac{(1-\alpha[z])}{\text{vf}}\right) \text{g Sin}[\theta]$$

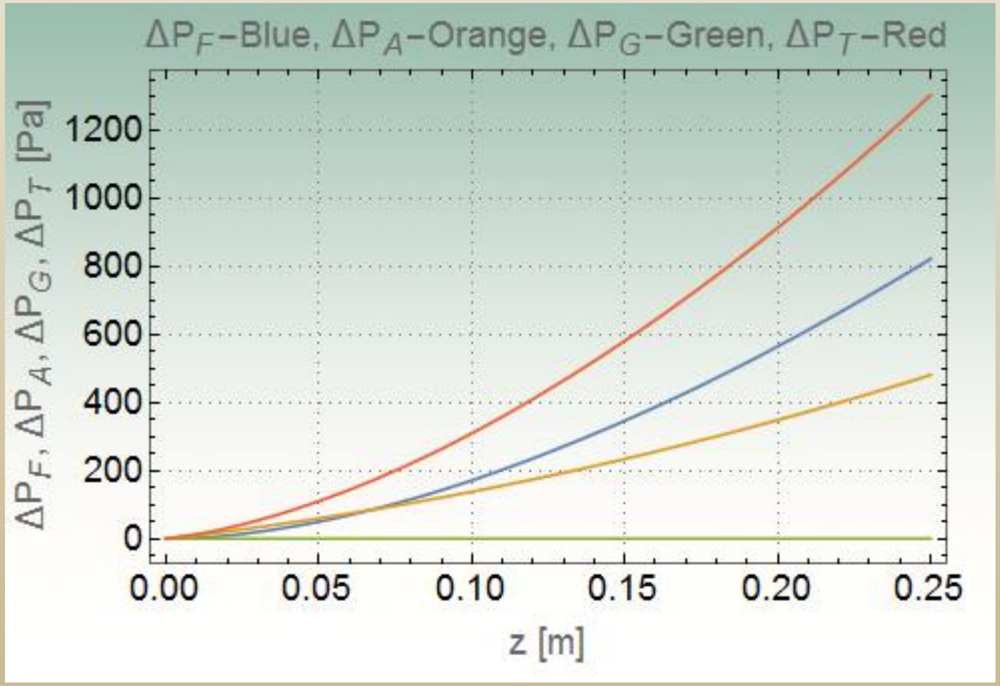
Integrated Pressure Drop, $\Delta P = \int_0^L -(dp/dz)\, dz$

$$\Delta\text{PF}[z\_] := \text{NIntegrate}[\text{dpdzF}[zz], \{zz, \text{zxe0}, z\}]$$

$$\Delta\text{PA}[z\_] := G^2\, \text{vf} \left(\frac{(\text{xe}[z])^2\, \text{vg}}{\alpha[z]\, \text{vf}} + \frac{(1-(\text{xe}[z]))^2}{(1-\alpha[z])} - 1\right)$$

$$\Delta\text{PG}[z\_] := \text{NIntegrate}[\text{dpdzG}[zz], \{zz, \text{zxe0}, z\}]$$

```
Plot[{ΔPF[z], ΔPA[z], ΔPG[z], ΔPF[z] + ΔPA[z] + ΔPG[z]},
 {z, zxe0 + .00001, intL}, Frame → True,
 FrameLabel → {"z [m]", "ΔPF, ΔPA, ΔPG, ΔPT [Pa]",
   "ΔPF-Blue, ΔPA-Orange, ΔPG-Green, ΔPT-Red", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```



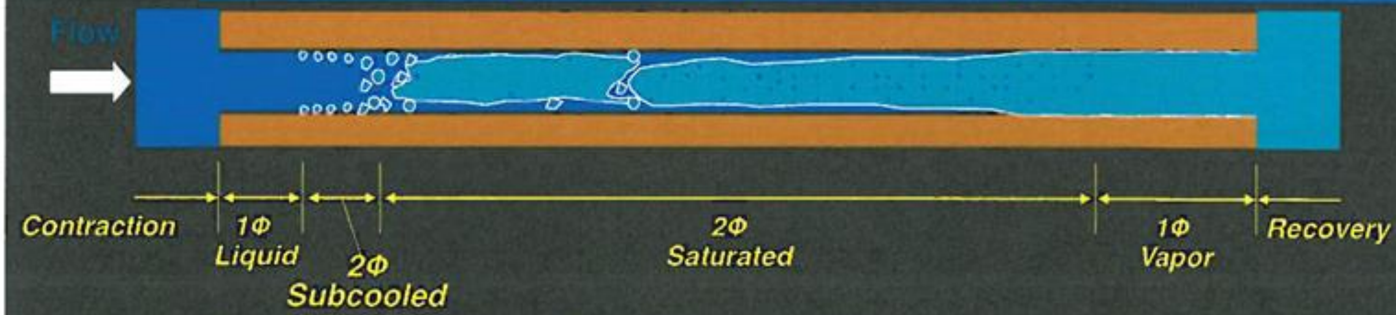
PRESSURE DROP IN SEPARATED FLOWS

Pressure Drop in Separated Flows-SFM with Mudawar's Universal Evaporating Flows Correlation

PRESSURE DROP IN SEPARATED FLOWS

PURDUE UNIVERSITY

Components of Complete Pressure Drop Model



A complete model must be able to tackle 11 components:

1. Inlet contraction
2. Developing laminar single-phase liquid region
3. Developing turbulent single-phase liquid region
4. Fully-developed laminar single-phase liquid region
5. Fully-developed turbulent single-phase liquid region
6. Subcooled two-phase region
7. Saturated two-phase region friction ← Dominant component
8. Saturated two-phase region acceleration
9. Laminar single-phase vapor region
10. Turbulent single-phase vapor region
11. Outlet pressure recovery

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More Recent Efforts

- Most published correlations for two-phase pressure drop recommended for relatively large tube diameters. Popular and successful correlations include those of Friedel (1979) and Müller-Steinhagen & Heck (1986)
- Small diameters crucial to reducing TCS mass in space systems and ensuring gravity independent evaporation and condensation
- New efforts undertaken at Purdue University Boiling and Two-Phase Flow Lab (PU-BTPFL) to derive universal correlations for small diameters (less than ~ 6 mm) by amassing published data for many fluids and over very broad ranges of operating conditions for:
  - Adiabatic and condensing flows
  - Evaporating flows
- The Purdue correlations are being tested against newly obtained microgravity data

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Limitations of Two-Phase Flow and Heat Transfer Correlations

- **One-phase: forced convection in a pipe:**

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \qquad \left\{ \begin{array}{l} Re > 10{,}000 \\ 0.6 < Pr < 160 \end{array} \right.$$

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

Very powerful correlation applicable to many fluids over very broad range of flow conditions

- **Two-phase: steam-water critical heat flux in a pipe:**

$$\frac{q''_m}{G h_{fg}} = f\left(\frac{\rho_f}{\rho_g}, \frac{G^2 L}{\sigma \rho_f}, \frac{c_{p,f}\,\Delta T_{sub}}{h_{fg}}, \frac{L}{D}, \frac{G}{\rho_f \sqrt{g D}}, \ldots\right)$$

$$\Pi_1 = f(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \ldots)$$

$$\left\{ \begin{array}{l} \Pi_{2,min} < \Pi_2 < \Pi_{2,max} \\ \Pi_{3,min} < \Pi_3 < \Pi_{3,max} \\ \Pi_{4,min} < \Pi_4 < \Pi_{4,max} \\ \Pi_{5,min} < \Pi_5 < \Pi_{5,max} \\ \Pi_{6,min} < \Pi_6 < \Pi_{6,max} \end{array} \right.$$

Simultaneously satisfying ranges for several parameters greatly limits overall usefulness of correlation ... Correlations cannot be extended with confidence to other fluids and/or beyond their validity range!

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PRESSURE DROP IN SEPARATED FLOWS

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Dimensionless Groups Employed by Various Investigators in Prediction of Two-Phase Pressure Gradient

Liquid- or gas-only Reynolds number:	$Re_{fo} = \frac{G D_h}{\mu_f}$, $Re_{go} = \frac{G D_h}{\mu_g}$	$\frac{\text{Inertia}}{\text{Viscous force}}$
Superficial liquid or gas Reynolds number:	$Re_f = \frac{G(1-x) D_h}{\mu_f}$, $Re_g = \frac{G x D_h}{\mu_g}$	$\frac{\text{Inertia}}{\text{Viscous force}}$
Density ratio:	$\frac{\rho_f}{\rho_g}$	$\frac{\text{Liquid density}}{\text{Vapor density}}$
Weber Number:	$We = \frac{G^2 D_h}{\rho_f \sigma}$	$\frac{\text{Inertia}}{\text{Surface tension force}}$
Capillary number:	$Ca = \frac{\mu_f G}{\rho_f \sigma}\left(= \frac{We}{Re_{fo}}\right)$	$\frac{\text{Viscous force}}{\text{Surface tension force}}$
Liquid- or gas-only Suratman number:	$Su_{fo} = \frac{\rho_f \sigma D_h}{\mu_f^2}\left(= \frac{Re_{fo}^2}{We}\right)$, $Su_{go} = \frac{\rho_g \sigma D_h}{\mu_g^2}\left(= \frac{Re_{go}^2}{We}\right)$	-
Froude number:	$Fr = \frac{G^2}{g D_h \rho_f^2}$	$\frac{\text{Inertia}}{\text{Body force}}$
Bond Number:	$Bd = \frac{g(\rho_f - \rho_g) D_h^2}{\sigma}$	$\frac{\text{Bouyancy force}}{\text{Surface tension force}}$
Confinement Number:	$N_{conf} = \sqrt{\frac{\sigma}{g(\rho_f - \rho_g) D_h^2}}\left(= \sqrt{\frac{1}{Bd}}\right)$	$\sqrt{\frac{\text{Surface tension force}}{\text{Body force}}}$
Galileo Number:	$Ga = \frac{\rho_f g (\rho_f - \rho_g) D_h^3}{\mu_f^2}$	-

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PRESSURE DROP IN SEPARATED FLOWS

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Pressure Drop in Saturated Two-Phase Flow Region

Two-phase pressure drop: $\boxed{\Delta p_{tp} = \Delta p_F + \Delta p_G + \Delta p_A}$

Accelerational pressure drop:

$$-\left(\frac{dp}{dz}\right)_A = G^2 \frac{d}{dz}\left[\frac{v_g x^2}{\alpha} + \frac{v_f (1-x)^2}{(1-\alpha)}\right] \quad \text{where} \quad \alpha = \left[1 + \left(\frac{1-x_e}{x_e}\right)\left(\frac{v_f}{v_g}\right)^{2/3}\right]^{-1}$$

(Zivi, 1964)

$$\begin{cases} \Delta p_A > 0 & \text{for boiling flows} \\ \Delta p_A < 0 & \text{for condensing flows} \end{cases}$$

Refrigeration

Gravitational pressure drop:

$$-\left(\frac{dp}{dz}\right)_G = \left[\alpha \rho_g + (1-\alpha)\rho_f\right] g \sin\phi$$

Frictional pressure drop:

Homogeneous Equilibrium Model (HEM)

$$-\left(\frac{dp}{dz}\right)_F = \frac{2 f_{tp} \bar{\rho} \bar{u}^2}{D_h} = \frac{2 f_{tp} v_f G^2}{D_h}\left(1 + x\frac{v_{fg}}{v_f}\right)$$

$f_{tp} = 16 Re_{tp}^{-1}$ for $Re_{tp} < 2{,}000$

$f_{tp} = 0.079 Re_{tp}^{-0.25}$ for $2{,}000 \le Re_{tp} < 20{,}000$

$f_{tp} = 0.046 Re_{tp}^{-0.2}$ for $Re_{tp} \ge 20{,}000$

where $Re_{tp} = \frac{G D_h}{\boxed{\mu_{tp}}}$

Separated Flow Model (SFM)

$$\left(\frac{dp}{dz}\right)_F = \left(\frac{dp}{dz}\right)_f \phi_f^2 \quad \text{where} \quad \phi_f^2 = 1 + \frac{\boxed{C}}{X} + \frac{1}{X^2}, \quad X^2 = \frac{(dp/dz)_f}{(dp/dz)_g}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2 f_f v_f G^2 (1-x)^2}{D_h}, \quad -\left(\frac{dp}{dz}\right)_g = \frac{2 f_g v_g G^2 x^2}{D_h}$$

$f_k = 16 Re_k^{-1}$ for $Re_k < 2{,}000$

$f_k = 0.079 Re_k^{-0.25}$ for $2{,}000 \le Re_k < 20{,}000$

$f_k = 0.046 Re_k^{-0.2}$ for $Re_k \ge 20{,}000$ where $k = f$ or g

Two-phase pressure drop: $\boxed{\Delta p_{tp} = \int_0^{L_{tp}} \left[-\left(\frac{dp}{dz}\right)_F - \left(\frac{dp}{dz}\right)_G - \left(\frac{dp}{dz}\right)_A\right] dz}$

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PRESSURE DROP IN SEPARATED FLOWS

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New PU-BTPFL Two-Phase Frictional Pressure Drop Correlation for Evaporating Flow in Small Diameter Tubes

Consolidated database:
2378 boiling pressure drop data points from 16 sources

- Working fluids:
  R12, R134a, R22, R245fa, R410A, FC-72, ammonia, CO2, water
- Hydraulic diameter:
  $0.349 < D_h < 5.35$ mm
- Mass velocity:
  $33 < G < 2738$ kg/m²s
- Liquid-only Reynolds number:
  $156 < Re_{fo} < 28{,}010$
- Superficial liquid Reynolds number:
  $0 < Re_f < 16{,}020$
- Superficial vapor (or gas) Reynolds number:
  $0 < Re_g < 199{,}500$
- Flow quality:
  $0 < x < 1$
- Reduced pressure:
  $0.005 < P_R < 0.78$

$$\left(\frac{dp}{dz}\right)_F = \left(\frac{dp}{dz}\right)_f \phi_f^2 \quad \text{where} \quad \phi_f^2 = 1 + \frac{\boxed{C}}{X} + \frac{1}{X^2}, \quad X^2 = \frac{(dp/dz)_f}{(dp/dz)_g}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2 f_f v_f G^2 (1-x)^2}{D_h}, \quad -\left(\frac{dp}{dz}\right)_g = \frac{2 f_g v_g G^2 x^2}{D_h}$$

$f_k = 16 Re_k^{-1}$ for $Re_k < 2{,}000$

$f_k = 0.079 Re_k^{-0.25}$ for $2{,}000 \le Re_k < 20{,}000$

$f_k = 0.046 Re_k^{-0.2}$ for $Re_k \ge 20{,}000$ where $k = f$ or g

for laminar flow in rectangular channel,

$$f_k Re_k = 24(1 - 1.3553\beta + 1.9467\beta^2 - 1.7012\beta^3 + 0.9564\beta^4 - 0.2537\beta^5)$$

$$Re_f = \frac{G(1-x)D_h}{\mu_f}, \quad Re_g = \frac{G x D_h}{\mu_g}, \quad Re_{fo} = \frac{G D_h}{\mu_f}, \quad Su_{go} = \frac{\rho_g \sigma D_h}{\mu_g^2}$$

$$C = C_{non-boiling}\left[1 + 530\, We_{fo}^{0.52}\left(Bo\,\frac{P_H}{P_F}\right)^{1.09}\right] \quad \text{for} \quad Re_f < 2000$$

$$C = C_{non-boiling}\left[1 + 60\, We_{fo}^{0.32}\left(Bo\,\frac{P_H}{P_F}\right)^{0.78}\right] \quad \text{for} \quad Re_f \ge 2000$$

where $We_{fo} = \dfrac{G^2 D_h}{\rho_f \sigma}$, $Bo = \dfrac{q''_H}{G\, h_{fg}}$

q''_H effective heat flux averaged over heated perimeter of channel

P_H heated perimeter of channel

P_F wetted perimeter of channel

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PRESSURE DROP IN SEPARATED FLOWS

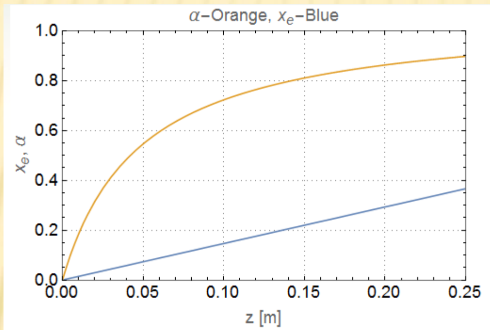
Example Problems

```
"Fluid is FC-72"

p = 2 (*bar*);
Cpf = 1136 (*J/kg.K*);
hfg = 87272 (*J/kg*);
vf = .0006515 (*m^3/kg*);
vg = .0387 (*m^3/kg*);
μf = 349.0×10^-6 (*kg/m.s*);
μg = 12.3×10^-6 (*kg/m.s*);
σ = .0062 (*N/m*);
q = 4.0×10^4 (*W/m^2*);
ΔTsub = 0 (*°C*);
g = 9.8 (*m.s^-2*);
θ = 0/180 π;
DD = .005 (*m*);
L = .25 (*m*);
G = 250 (*kg/m^2.s*);
W = G π (DD^2/4);

A = π DD^2/4 (*m^2*); peri = π DD (*m*);
DF = 4 A/peri (*m*);
vfg = vg - vf;
ReyNum = G DF / μf;
ReyNumg = G DF / μg;
BoNum = q/(G hfg)
WeNumf0 = G^2 DF vf/σ

xe[z_] := -Cpf ΔTsub/hfg + π DD q/(W hfg) z
xe0 = xe[0];
xel = xe[L];

zxe0 = W Cpf ΔTsub/(π DD q); L1ph = zxe0;
zxe1 = hfg W/(π DD q) + Cpf ΔTsub W/(π DD q); L2ph = zxe1;
If[L2ph < L, intL = L2ph, intL = L];
```

Void Fraction and Quality (Zivi, 1964)

```
α[z_] := (1 + (1 - xe[z])/xe[z] (vf/vg)^(2/3))^-1
PF[z_] := π DD (1 - α[z]); PF[z_] := π DD;
PH = π DD
0.015708
```

```
Plot[{xe[z], α[z]}, {z, zxe0 + .00001, intL}, Frame → True,
  FrameLabel → {"z [m]", "xe, α", "α-Orange, xe-Blue", ""},
  LabelStyle → (FontSize → 18), FrameTicks → Automatic,
  FrameTicksStyle → Black, GridLines → Automatic,
  GridLinesStyle → Directive[Dotted, Gray],
  PlotRange → {{0, L}, {0, 1}}]
```



Friction Factors on the Liquid and Gas Sides

```
ff[z_] :=
 Piecewise[{{16 (G (1 - xe[z]) DD/μf)^-1, (G (1 - xe[z]) DD/μf) < 2000},
   {.079 (G (1 - xe[z]) DD/μf)^-.25, 2000 < (G (1 - xe[z]) DD/μf) < 20000},
   {.046 (G (1 - xe[z]) DD/μf)^-.2, (G (1 - xe[z]) DD/μf) > 20000}}]
fg[z_] :=
 Piecewise[{{16 (G (xe[z]) DD/μg)^-1, (G (xe[z]) DD/μg) < 2000},
   {.079 (G (xe[z]) DD/μg)^-.25, 2000 < (G (xe[z]) DD/μg) < 20000},
   {.046 (G (xe[z]) DD/μg)^-.2, (G (xe[z]) DD/μg) > 20000}}]
```

Constant $C_{\text{Non-Boiling}}$

```
CC[z_] :=
 Piecewise[{{5, (G (1 - xe[z]) DD/μf) < 2000 && (G (xe[z]) DD/μg) < 2000},
   {12, (G (1 - xe[z]) DD/μf) < 2000 && 2000 < (G (xe[z]) DD/μg)},
   {10, 2000 < (G (1 - xe[z]) DD/μf) && (G (xe[z]) DD/μg) < 2000},
   {20, 2000 < (G (1 - xe[z]) DD/μf) && 2000 < (G (xe[z]) DD/μg)}}]
```

Constant C_{Boiling}

```
CCM[z_] :=
 Piecewise[
  {{CC[z] (1 + 530 WeNumf0^.52 (BoNum PH / PF[z])^1.09),
    (G (1 - xe[z]) DD/μf) < 2000},
   {CC[z] (1 + 60 WeNumf0^.32 (BoNum PH / PF[z])^.78),
    (G (1 - xe[z]) DD/μf) > 2000}}]
```

Frictional, Acceleration and Gravitational Pressure Gradients

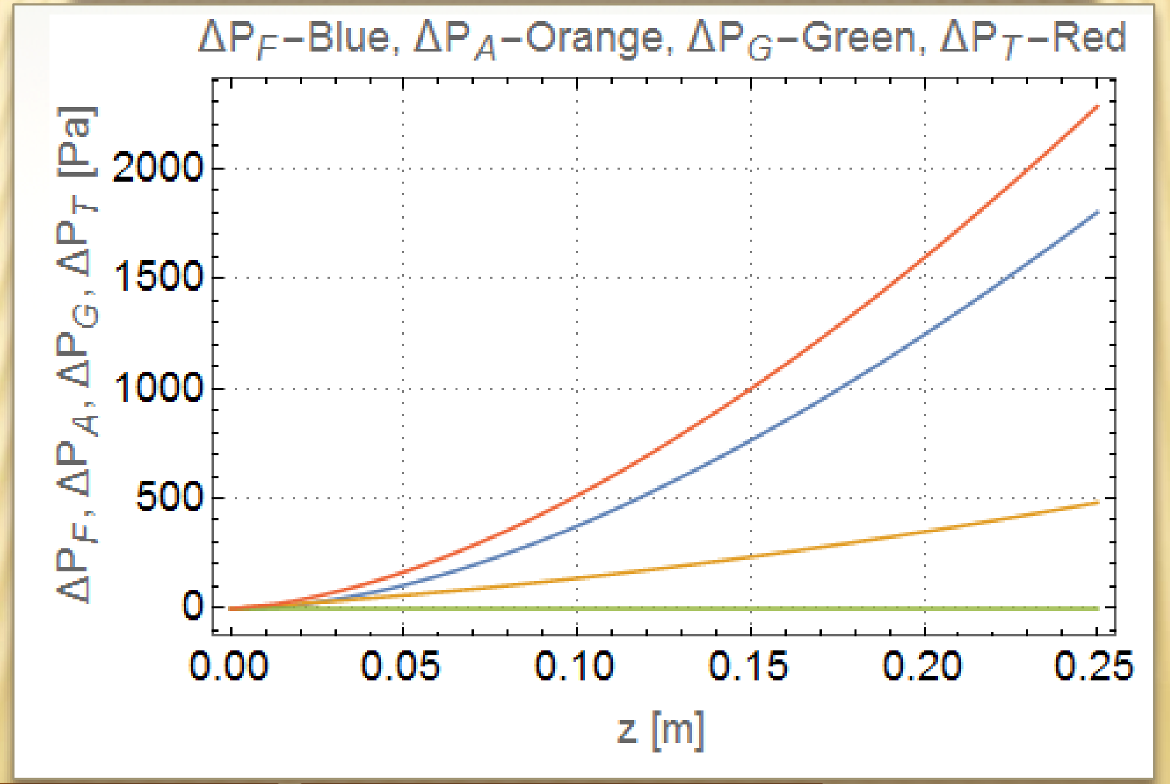
```
dpdzF[z_] :=
 1/DD 2 G^2 vf ff[z] (1 - xe[z])^2
  (1 + CCM[z]/Sqrt[(vf ff[z] (1-xe[z])^2)/(vg fg[z] xe[z]^2)] + (vg fg[z] xe[z]^2)/(vf ff[z] (1 - xe[z])^2))
dpdzA[z_] := G^2 vf ((xe[z])^2 vg/(α[z] vf) + (1 - (xe[z]))^2/(1 - α[z]) - 1)
dpdzG[z_] := (α[z]/vg + (1 - α[z])/vf) g Sin[θ]
```

Integrated Pressure Drop, $\Delta P = \int_o^L -(dp/dz)\, dz$

```
ΔPF[z_] := NIntegrate[dpdzF[zz], {zz, xe0, z}]
ΔPA[z_] := G^2 vf ((xe[z])^2 vg/(α[z] vf) + (1 - (xe[z]))^2/(1 - α[z]) - 1)
ΔPG[z_] := NIntegrate[dpdzG[zz], {zz, zxe0, z}]
```

PRESSURE DROP IN SEPARATED FLOWS

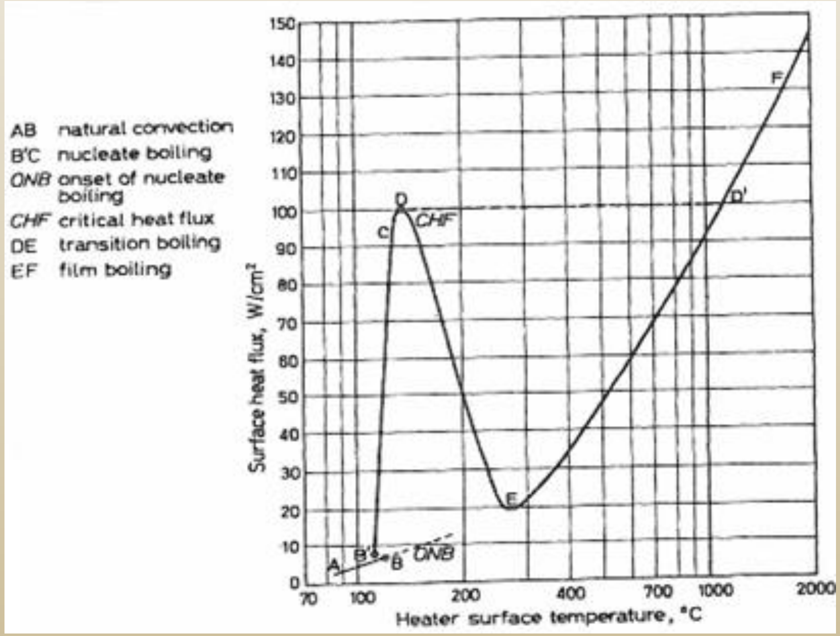
```
Plot[{ΔPF[z], ΔPA[z], ΔPG[z], ΔPF[z] + ΔPA[z] + ΔPG[z]},
 {z, zxe0 + .00001, intL}, Frame → True,
 FrameLabel → {"z [m]", "ΔPF, ΔPA, ΔPG, ΔPT [Pa]",
   "ΔPF-Blue, ΔPA-Orange, ΔPG-Green, ΔPT-Red", ""},
 LabelStyle → (FontSize → 18), FrameTicks → Automatic,
 FrameTicksStyle → Black, GridLines → Automatic,
 GridLinesStyle → Directive[Dotted, Gray], PlotRange → All]
```



Boiling and Condensation Heat Transfer

BOILING AND CONDENSATION HEAT TRANSFER

Pool Boiling



Incipient Boiling Heat Flux

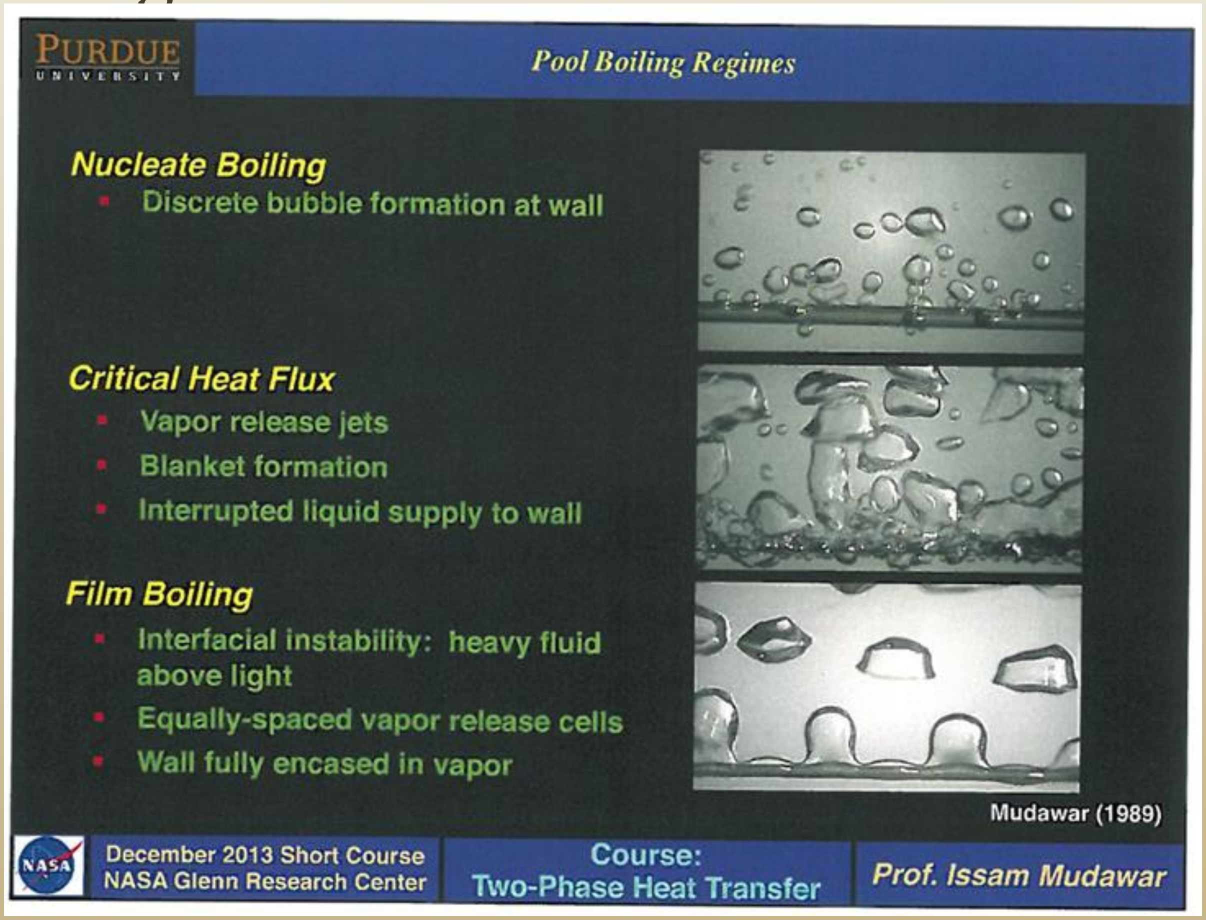
$$q''_i = \frac{8\,\sigma\,T_{sat}\,v_{fg}\,h^2}{k_f\,h_{fg}}$$

Superheat required at the boiling inception

$$\boxed{T_{wi} - T_{sat} = \frac{1}{\Gamma} = \frac{8\,\sigma\,T_{sat}\,v_{fg}\,h}{k_f\,h_{fg}}}$$

BOILING AND CONDENSATION HEAT TRANSFER

- Pool Boiling



BOILING AND CONDENSATION HEAT TRANSFER

- Pool Boiling

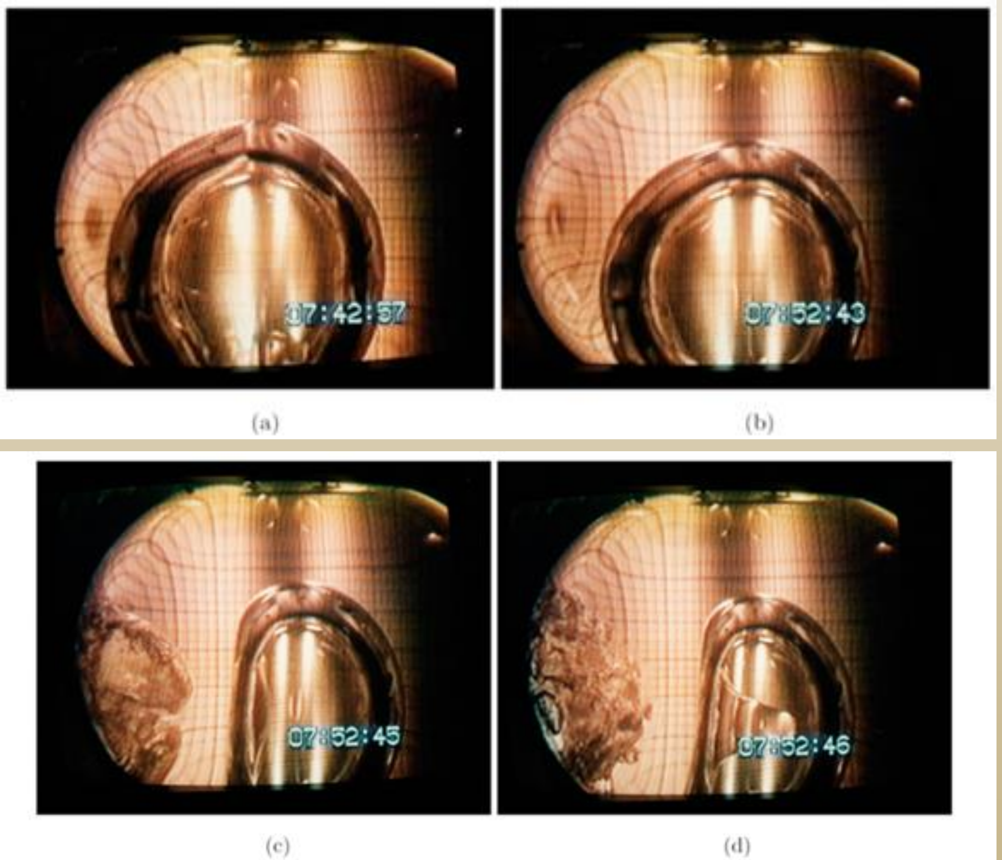


Figure 4. Explosive boiling from heater A and subsequent events (run 13). (a) Initial liquid-vapor configuration: elapsed heating time, 0 s. (b) Liquid-vapor configuration: elapsed heating time, 9 min and 46 s. (c) Initiation of explosive boiling: elapsed heating time, 9 min and 48 s; wall superheat, 17.9 deg C. (d) Growth of vapor mass and violent bulk liquid motion: elapsed heating time, 9 min and 49 s.

Incipient Boiling Heat Flux

$$q''_i = \frac{8\,\sigma\,T_{sat}\,v_{fg}\,h^2}{k_f\,h_{fg}}$$

BOILING AND CONDENSATION HEAT TRANSFER

- Nucleate Boiling

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Nucleate Boiling: Rohsenow Model

Therefore,

$$\boxed{q'' = \mu_f h_{fg} \left[\frac{g(\rho_f - \rho_g)}{\sigma} \right]^{1/2} \left[\frac{c_{p,f}(T_w - T_{sat})}{C_{sf}\, h_{fg}\, Pr_f^s} \right]^3}$$

C_{sf} and s depend on fluid, surface material & surface finish

Rohsenow correlation for saturated nucleate pool boiling

Overall, Rohsenow correlation shows it is difficult to develop a single universal nucleate boiling relation for all fluids, surface materials and surface finishes

Realistic representation and relatively simple formulation has made this correlation the most popular tool for predicting saturated nucleate pool boiling heat transfer

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BOILING AND CONDENSATION HEAT TRANSFER

Nucleate Boiling

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Nucleate Boiling: Rohsenow Model

Examples of empirical values for C_{sf} and s

Fluid	Wall Material	Surface Finish	C_{sf}	s
Water	Copper	Rough	0.0068	1.0
		Polished	0.0130	1.0
n-Pentane	Copper	Polished	0.0154	1.7
		Lapped	0.0049	1.7

- **Lower $C_{sf} \Rightarrow$ lower superheat for rougher surface**
- **$s = 1.0$ for water**

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BOILING AND CONDENSATION HEAT TRANSFER

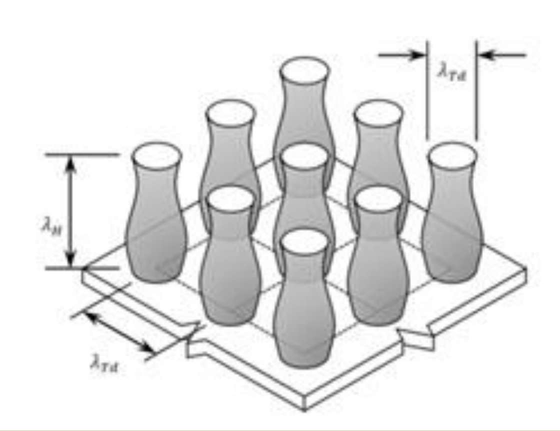
Pool Boiling Critical Heat Flux

Pool Boiling Critical Heat Flux (CHF)

at Earth's Gravity

$$\boxed{q''_{max} = 0.131\, \rho_g h_{fg} \left[\frac{\sigma g (\rho_f - \rho_g)}{\rho_g^2} \right]^{1/4}} \quad \text{for } \rho_g \ll \rho_f$$

Zuber CHF model

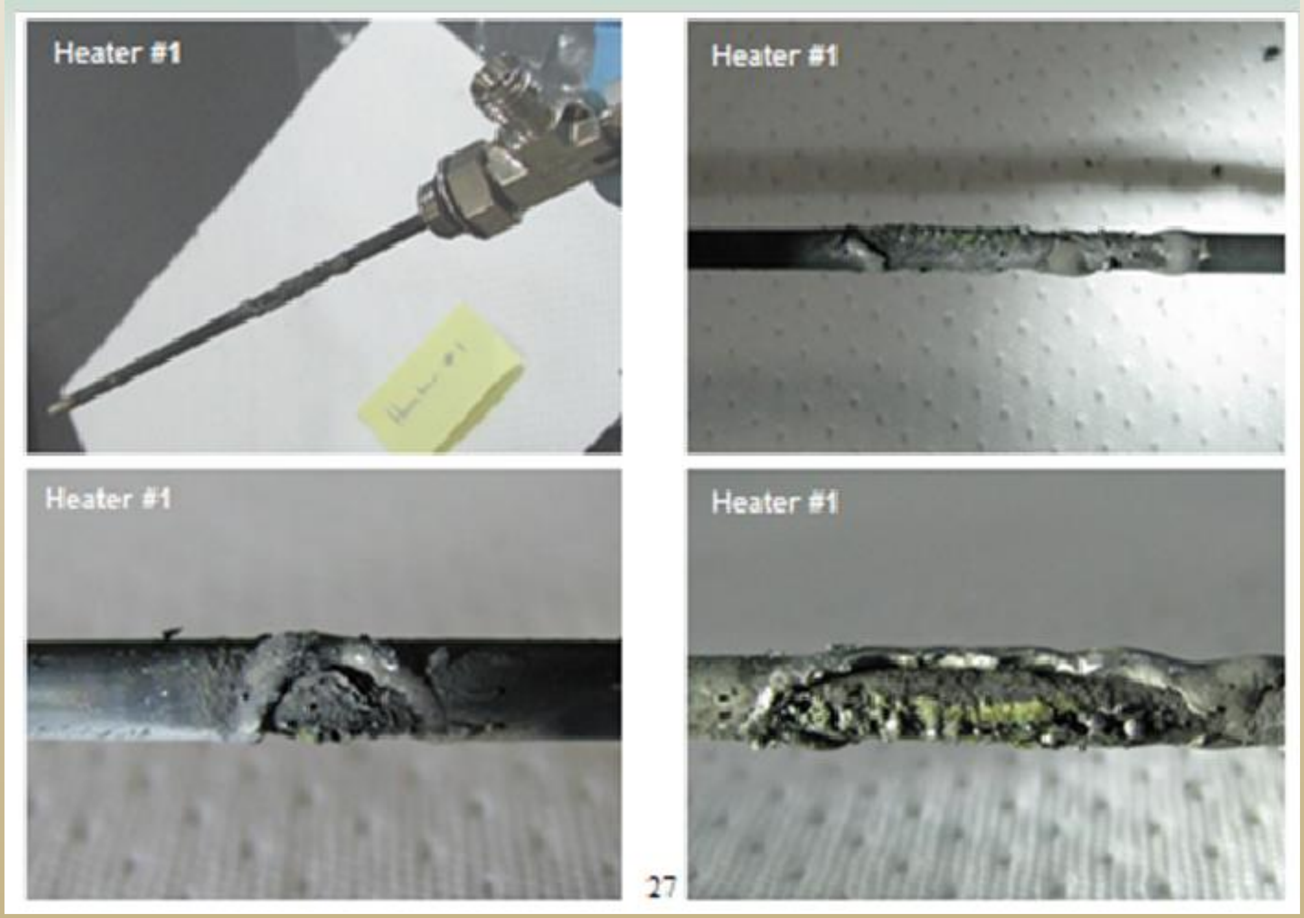


Critical Heat Flux (CHF) for FC-72 ~ 18 W/cm²
CHF for water ~ 100 W/cm²

What is the pool boiling Critical Heat Flux (CHF) as g approaches 0 (microgravity environment)?
This is what happens in a microgravity experiment with nPFH for a heat flux of about 4 W/cm²

BOILING AND CONDENSATION HEAT TRANSFER

Heater Burnout in Microgravity

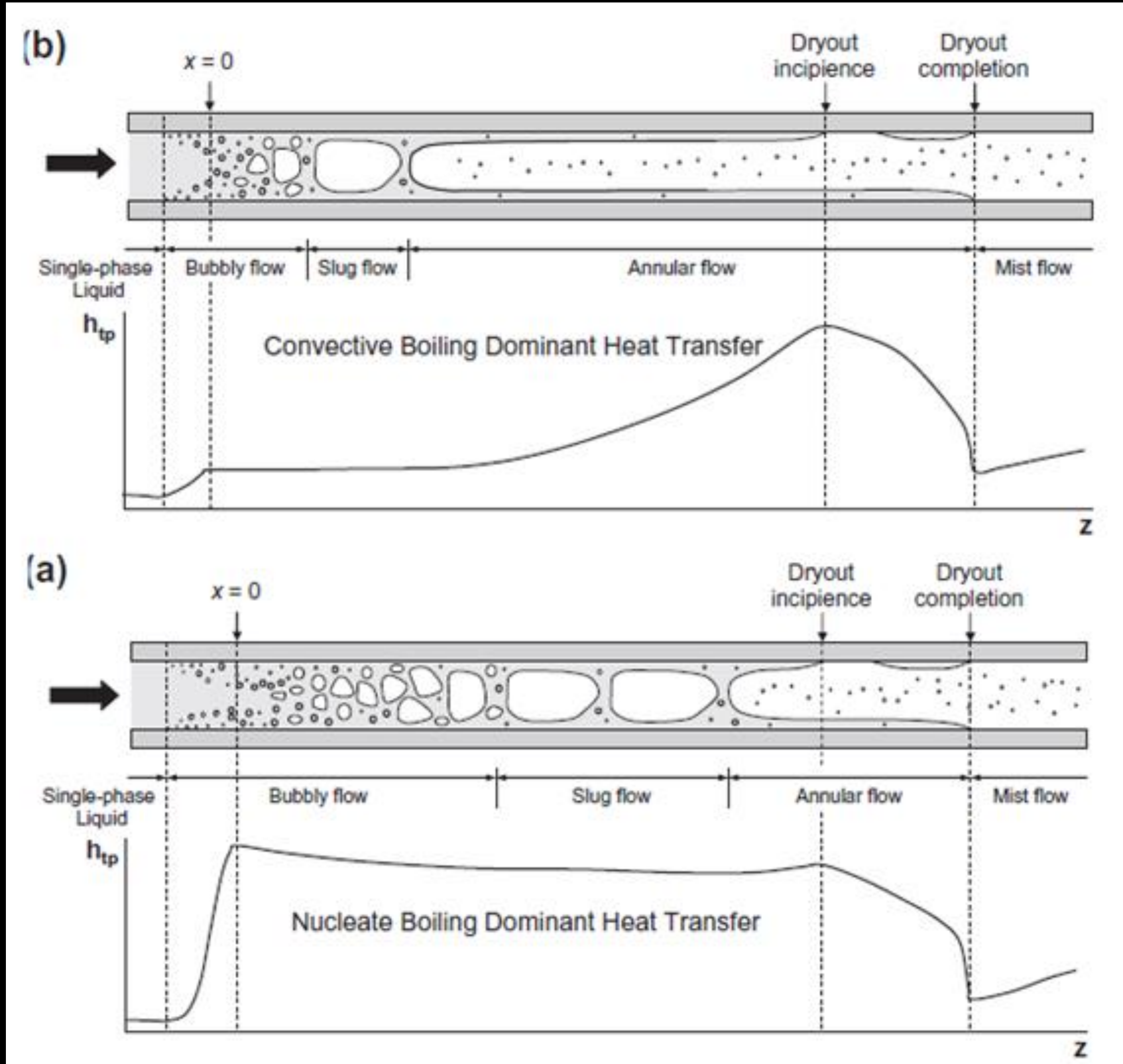


FLOW BOILING HEAT TRANSFER

- Flow Boiling Heat Transfer

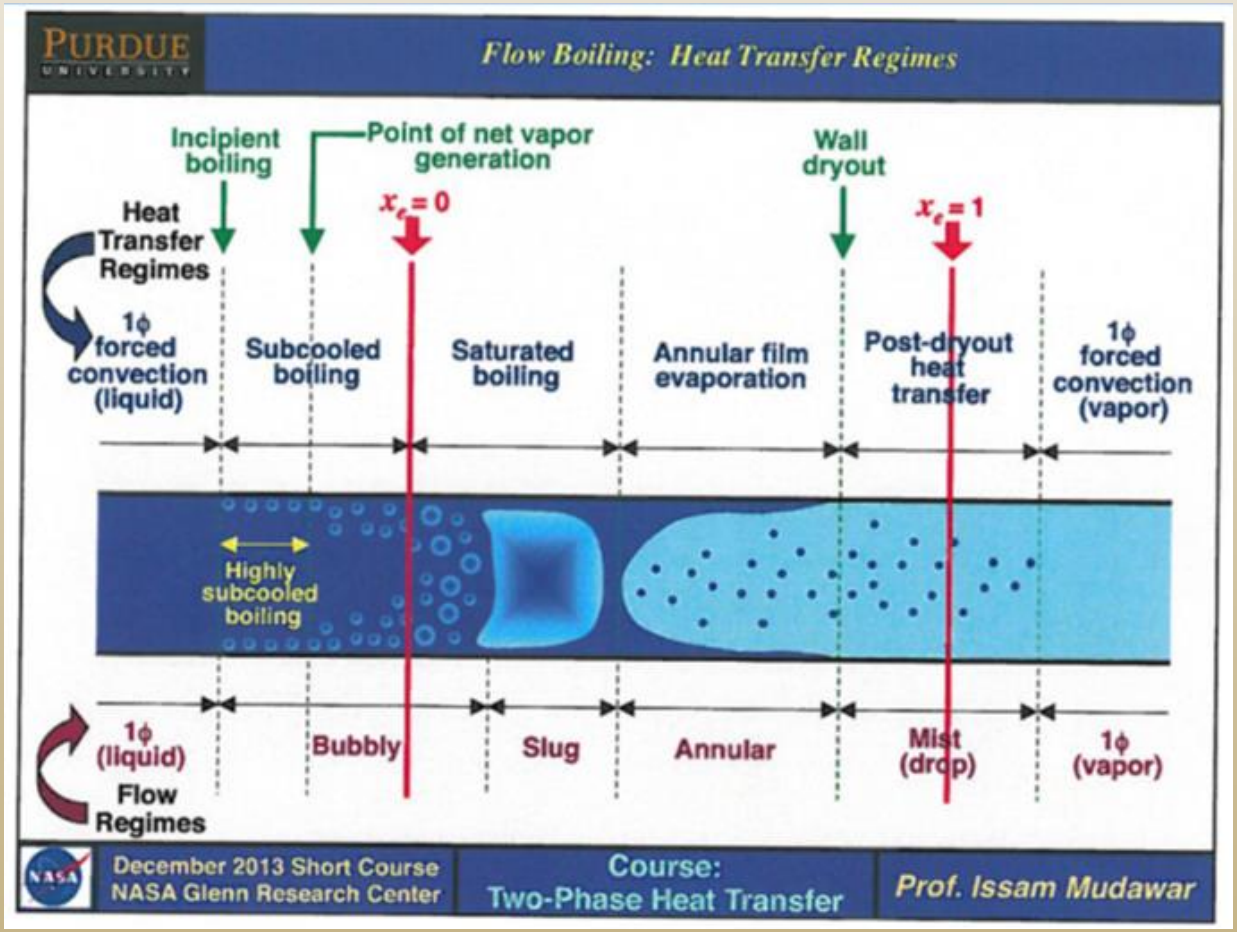
FLOW BOILING

- Depiction of Convective Boiling Dominant Heat Transfer and Nucleate Boiling Dominant heat Transfer



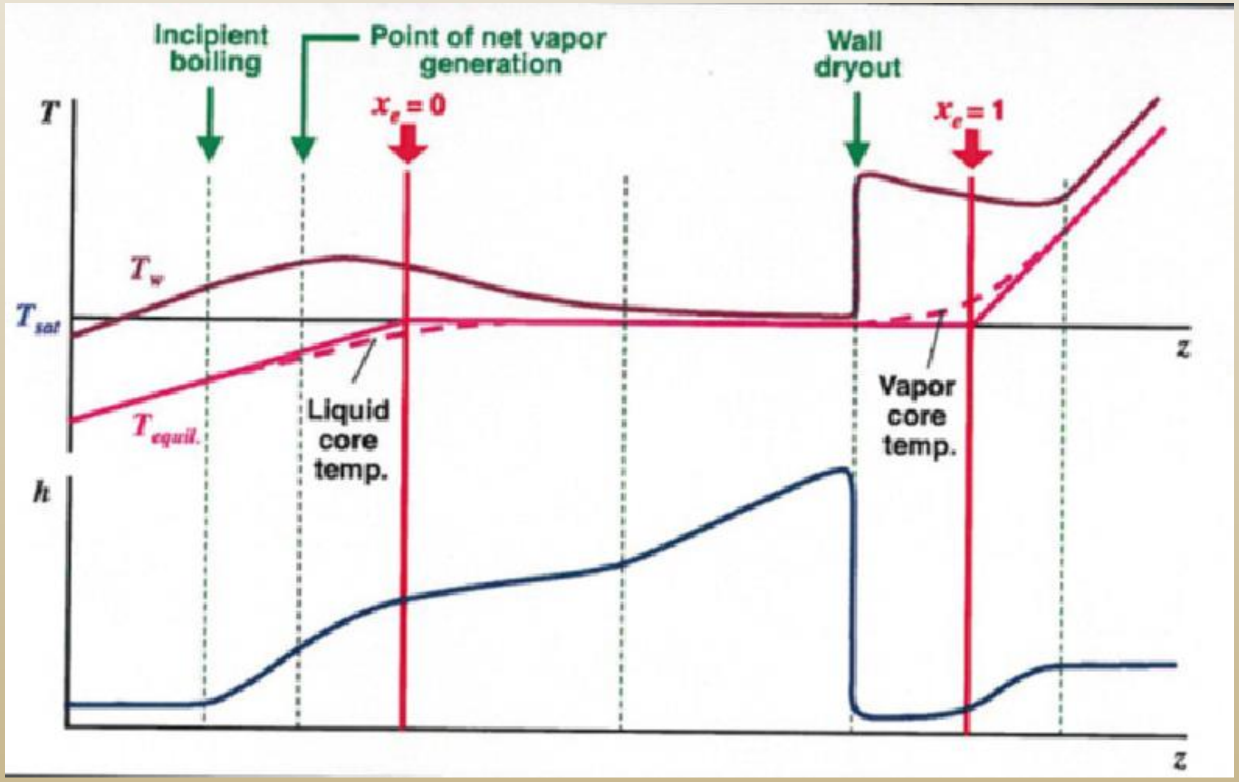
FLOW BOILING HEAT TRANSFER

Flow Boiling Heat Transfer



FLOW BOILING HEAT TRANSFER

- Flow Boiling Heat Transfer



FLOW BOILING HEAT TRANSFER

- Flow Boiling Heat Transfer

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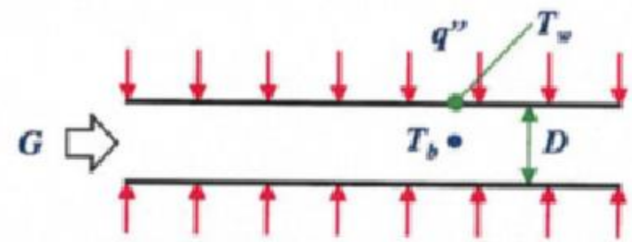
Flow Boiling: Heat Transfer Correlations

1. Subcooled and Low-Quality Saturated Boiling Region ($x_e < 0.05$)

(Bjorge, Hall & Rohsenow, 1982)



Single-phase forced convection:

$$q''_{FC} = h_{FC}\left(T_w - T_b\right) = h_{FC}\left[\left(T_w - T_{sat}\right) + \left(T_{sat} - T_b\right)\right]$$

where $$\frac{h_{FC}\,D}{k_f} = 0.023\left(\frac{G\,D}{\mu_f}\right)^{0.8} Pr_f^{1/3}$$

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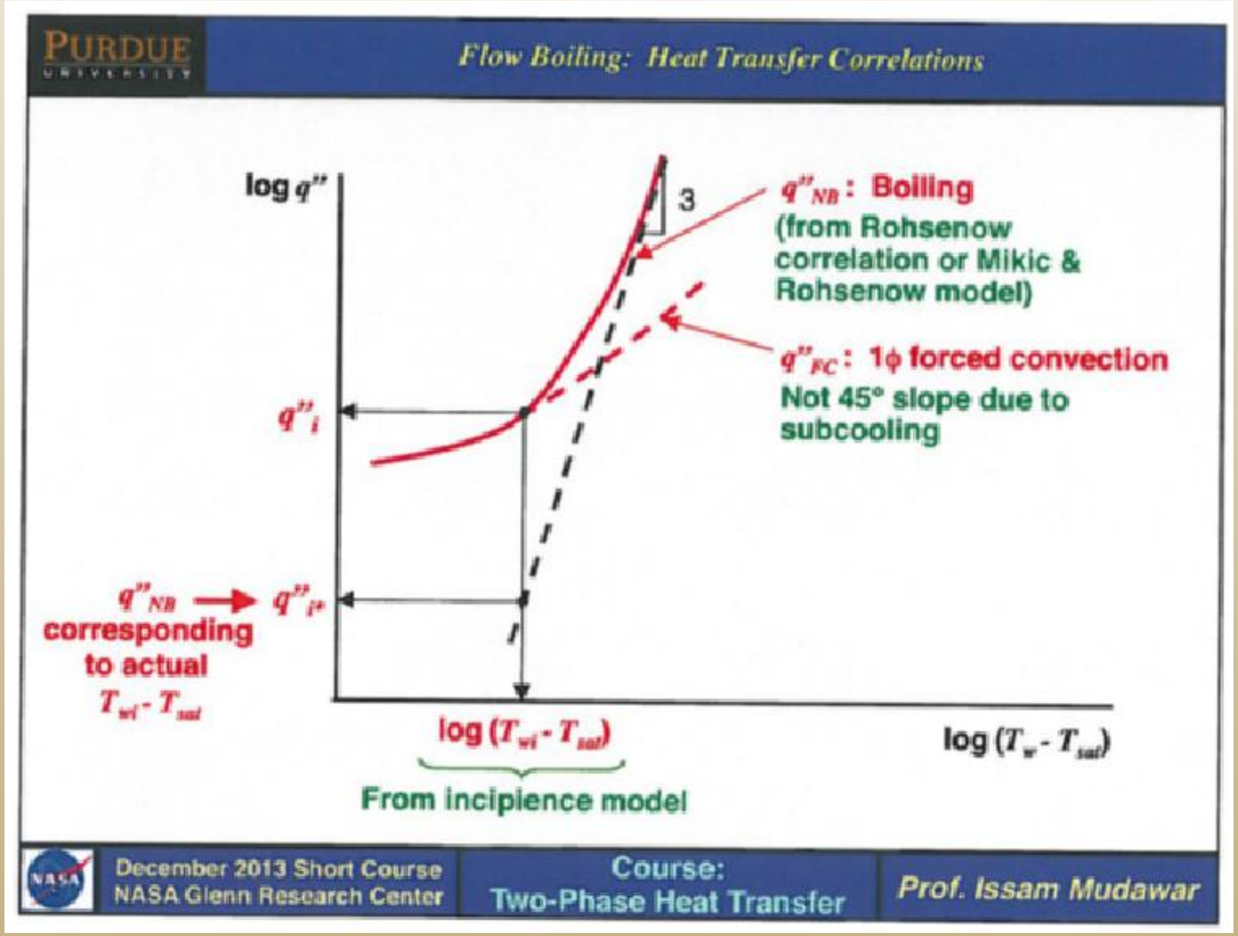
FLOW BOILING HEAT TRANSFER

- Flow Boiling Heat Transfer



- Flow Boiling Heat Transfer
- Flow Boiling Heat Transfer in Subcooled Region xe<.05

$$q'' = \left[q''^2_{FC} + \Big(\overbrace{q''_{NB} - q''_{i*}}^{\text{Excess above forced convection due to boiling}}\Big)^2\right]^{1/2} = \left[q''^2_{FC} + q''^2_{NB}\left(1 - \frac{q''_{i*}}{q''_{NB}}\right)^2\right]^{1/2}$$

$$\Rightarrow \boxed{q'' = \left[q''^2_{FC} + q''^2_{NB}\left\{1 - \left(\frac{T_{wi} - T_{sat}}{T_w - T_{sat}}\right)^3\right\}^2\right]^{1/2}}$$

$$q'' = f(T_w) \quad \text{or} \quad T_w = f(q'')$$

Flow Boiling Heat Transfer

Saturated Boiling Region ($x_e > 0.05$)

Chen (1963) ASME Paper 63-HT-34

$$\boxed{\frac{q''}{T_w - T_{sat}} = h = S\,h_{NB} + F\,h_{FC}}$$

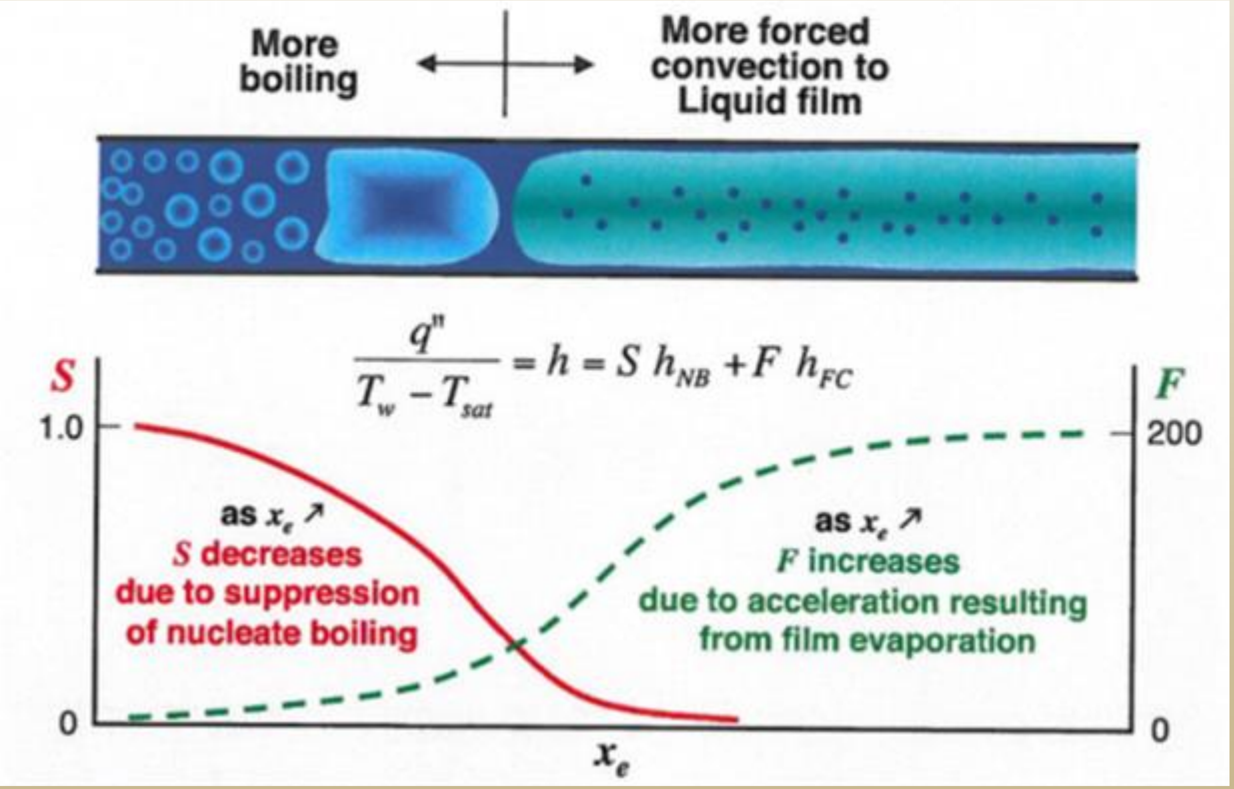
h_{NB} : **Nucleate boiling**

S : **Boiling suppression factor**

h_{FC} : **Forced convection to liquid film**

F : **2φ heat transfer multiplier**

Flow Boiling Heat Transfer



FLOW BOILING HEAT TRANSFER

- Flow Boiling Heat Transfer

where

$$\boxed{h_{NB} = 0.00122\left(\frac{k_f^{0.79}\, c_{p,f}^{0.45}\, \rho_f^{0.49}}{\sigma^{0.5}\, \mu_f^{0.29}\, h_{fg}^{0.24}\, \rho_g^{0.24}}\right)\left(T_w - T_{sat}\right)^{0.24}\left(p_{sat}\big|_{T_w} - p_{sat}\big|_{T_{sat}}\right)^{0.75}}$$

Based on Forster & Zuber correlation (1955) for nucleate pool boiling

and $$\boxed{\frac{h_{FC}\, D}{k_f} = 0.023\, Re_D^{0.8}\, Pr_f^{1/3}}$$

FLOW BOILING HEAT TRANSFER

- Flow Boiling Heat Transfer

Chen's correlation: Calculation procedure

- Calculate the Martinelli parameter for the flow
- Evaluate the empirical function F
- Calculate the single phase heat transfer coefficient
- Calculate the two-phase Reynolds number Re_{TP}
- Evaluate the empirical factor S
- Calculate the nucleate boiling heat transfer coefficient
- Calculate two-phase heat transfer coefficient h_{TP}
- Calculate $q = h_{TP}\,\Delta T_{sat}$

FLOW BOILING HEAT TRANSFER

- Examples of Flow Boiling Heat Transfer Coefficient Prediction

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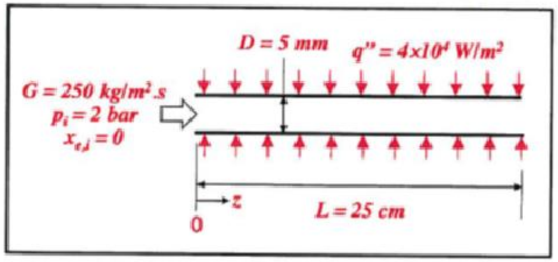
Numerical Example 4: Determination of Flow Boiling Heat Transfer Coefficient using Chen (1966) Correlation

Saturated FC-72 ($x_e = 0$) at mass velocity of $G = 250$ kg/m^2.s and inlet pressure of $p_i = 2$ bar enters a horizontal circular tube of diameter $D = 5$ mm and length $L = 25$ cm, where it is subjected to a constant heat flux of $q'' = 4\times10^4$ W/m^2.
Assuming constant thermophysical properties and using five Δz increments along the flow direction, use the Chen (1966) correlation to determine the following:

(a) $x_e(z), x_{e,L}$

(b) $h(z)$

(c) $\overline{h}(z)$



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FLOW BOILING HEAT TRANSFER

Flow Boiling Heat Transfer

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Numerical Example 4: Determination of Flow Boiling Heat Transfer Coefficient using Chen (1966) Correlation

Solution:

Thermophysical properties of FC-72 at p = 2 bar: $c_{p,f}$ = 1136 J/kg.K, h_{fg} = 87272 J/kg, v_f = 0.0006515 m³/kg, v_g = 0.0387 m³/kg, μ_f = 349.0×10⁻⁶ kg/m.s, μ_g = 12.3×10⁻⁶ kg/m.s, σ = 0.0062 N/m, k_f = 0.0514 W/m.K, Pr_f = 7.7212, p_{crit} = 1830 kPa

(a)

$$x_e = \frac{(\pi D)\, q''}{G\left(\frac{\pi D^2}{4}\right) h_{fg}}\, z = \frac{(\pi \times 0.005\ m) \times 4 \times 10^4\ W/m^2}{250\ kg/m^2 s \times \left[\frac{\pi \times (0.005\ m)^2}{4}\right] \times 87272\ J/kg}\, z = \boxed{1.467\, z}$$

$$x_{e,L} = 1.467 \frac{1}{m} \times 0.25\ m = \boxed{0.367}$$

(b)

Chen (1966): $h = E\, h_{sp} + S\, h_{nb}$

$$h_{sp} = 0.023 Re_f^{0.8} Pr_f^{0.4} \frac{k_f}{D}$$

$$h_{nb} = 0.00122 \left(\frac{k_f^{0.79} c_{p,f}^{0.45} v_g^{0.24}}{\sigma^{0.5} \mu_f^{0.29} h_{fg}^{0.24} v_f^{0.49}} \right) \Delta T_{sat}^{0.24} \Delta P_{sat}^{0.75}$$

$$E = \left(1 + \frac{1}{X_{tt}^{0.5}}\right)^{1.78}, \quad S = 0.9622 - 0.5822 \tan^{-1}\left\{ \frac{Re_f E^{1.25}}{6.18 \times 10^4} \right\}$$

where

$$X_{tt} = \left(\frac{\mu_f}{\mu_g}\right)^{0.1} \left(\frac{1 - x_e}{x_e}\right)^{0.9} \left(\frac{\rho_g}{\rho_f}\right)^{0.5}, \quad Re_f = \frac{G(1 - x_e) D}{\mu_f}$$

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FLOW BOILING HEAT TRANSFER

Flow Boiling Heat Transfer

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Numerical Example 4: Determination of Flow Boiling Heat Transfer Coefficient using Chen (1966) Correlation

$\Delta T_{sat} = T_w - T_{sat}$

$\Delta P_{sat} = P_w (P_{sat}\ at\ T_w) - P_{sat}$

Wall temperature is numerically calculated using $T_w = T_{sat} + q'' / h$

Node #	1	2	3	4	5
x_e	0.073	0.147	0.220	0.293	0.367
h_{sp}	350.9	328.5	305.7	282.5	258.8
E	2.708	3.630	4.502	5.407	6.396
S	0.855	0.821	0.794	0.773	0.754
ΔT_{sat}	11.25	11.00	10.82	10.66	10.53
ΔP_{sat}	7.27×10^4	7.10×10^4	6.96×10^4	6.85×10^4	6.76×10^4
h_{nb}	3049	2977	2922	2878	2840
h	3556	3636	3698	3751	3797

(c) $\bar{h} = \frac{1}{L}\int_0^L h(z)dz = \boxed{3688\ W/m^2K}$

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FLOW BOILING HEAT TRANSFER

Flow Boiling Heat Transfer

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Numerical Example 3: Determination of Flow Boiling Heat Transfer Coefficient using Kim and Mudawar (2013) Correlation

Solution:

Thermophysical properties of FC-72 at p = 2 bar: $c_{p,f}$ = 1136 J/kg.K, h_{fg} = 87272 J/kg, v_f = 0.0006515 m³/kg, v_g = 0.0387 m³/kg, μ_f = 349.0x10⁻⁶ kg/m.s, μ_g = 12.3x10⁻⁶ kg/m.s, σ = 0.0062 N/m, k_f = 0.0514 W/m.K, Pr_f = 7.7212, p_{crit} = 1830 kPa

(a)
$$x_e = \frac{(\pi D)\, q''}{G\left(\frac{\pi D^2}{4}\right) h_{fg}}\, z = \frac{(\pi \times 0.005\ m) \times 4 \times 10^4\ W/m^2}{250\ kg/m^2 s \times \left[\frac{\pi \times (0.005\ m)^2}{4}\right] \times 87272\ J/kg}\, z = \boxed{1.467\ z}$$

$$x_{e,L} = 1.467 \frac{1}{m} \times 0.25\ m = \boxed{0.367}$$

(b) Kim and Mudawar (2013): $h = \left(h_{nb}^2 + h_{cb}^2\right)^{0.5}$

For nucleate boiling dominant regime :

$$h_{nb} = \left[2345\left(Bo\frac{P_H}{P_F}\right)^{0.70} P_R^{0.38}\left(1-x_e\right)^{-0.51}\right]\left(0.023\, Re_f^{0.8}\, Pr_f^{0.4}\, \frac{k_f}{D_h}\right)$$

For convective boiling dominant regime :

$$h_{cb} = \left[5.2\left(Bo\frac{P_H}{P_F}\right)^{0.08} We_{fo}^{-0.54} + 3.5\left(\frac{1}{X_{tt}}\right)^{0.94}\left(\frac{\rho_g}{\rho_f}\right)^{0.25}\right]\left(0.023\, Re_f^{0.8}\, Pr_f^{0.4}\, \frac{k_f}{D_h}\right)$$

where

$$Bo = \frac{q''_H}{G\, h_{fg}}\ ,\quad P_R = \frac{P}{P_{crit}}\ ,\quad Re_f = \frac{G\left(1-x_e\right) D}{\mu_f}\ ,\quad We_{fo} = \frac{G^2 D}{\rho_f\, \sigma}\ ,\quad X_{tt} = \left(\frac{\mu_f}{\mu_g}\right)^{0.1}\left(\frac{1-x_e}{x_e}\right)^{0.9}\left(\frac{\rho_g}{\rho_f}\right)^{0.5}$$

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FLOW BOILING HEAT TRANSFER

Flow Boiling Heat Transfer

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Numerical Example 3: Determination of Flow Boiling Heat Transfer Coefficient using Kim and Mudawar (2013) Correlation

q''_H : effective heat flux averaged over heated perimeter of channel

P_H: heated perimeter of channel

P_F: wetted perimeter of channel

Node #	1	2	3	4	5
x_e	0.073	0.147	0.220	0.293	0.367
Bo	0.0018	0.0018	0.0018	0.0018	0.0018
P_H/P_F	1	1	1	1	1
P_R	0.1093	0.1093	0.1093	0.1093	0.1093
We_{fo}	32.861	32.861	32.861	32.861	32.861
h_{nb}	4479	4373	4260	4140	4011
h_{cb}	425	621	803	977	1146
h	4499	4417	4335	4254	4171

Large values of h_{nb}/h_{cb} indicate that nucleate boiling is dominant heat transfer regime

(c) $\bar{h} = \frac{1}{L}\int_0^L h(z)dz = \boxed{4335\ W/m^2K}$

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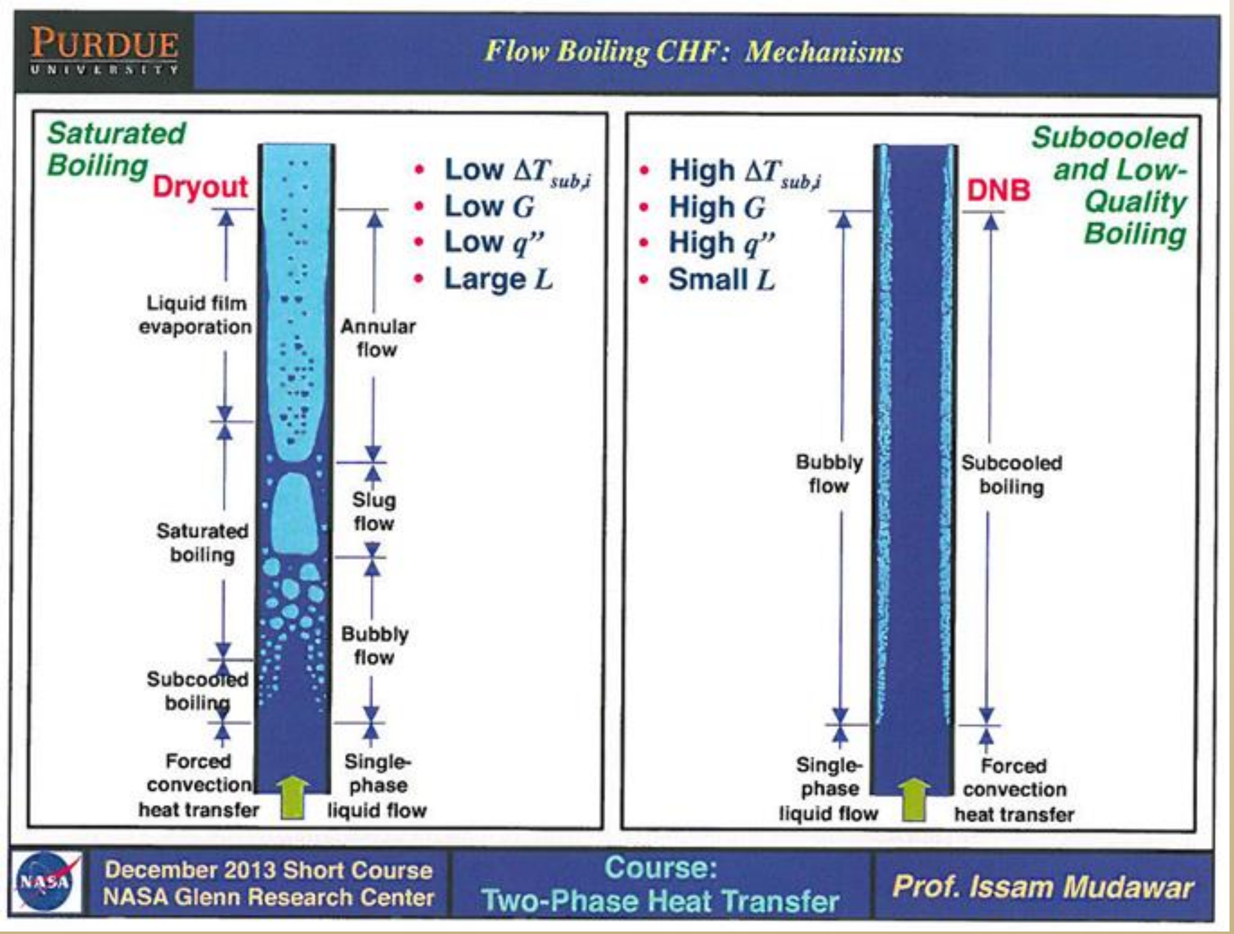
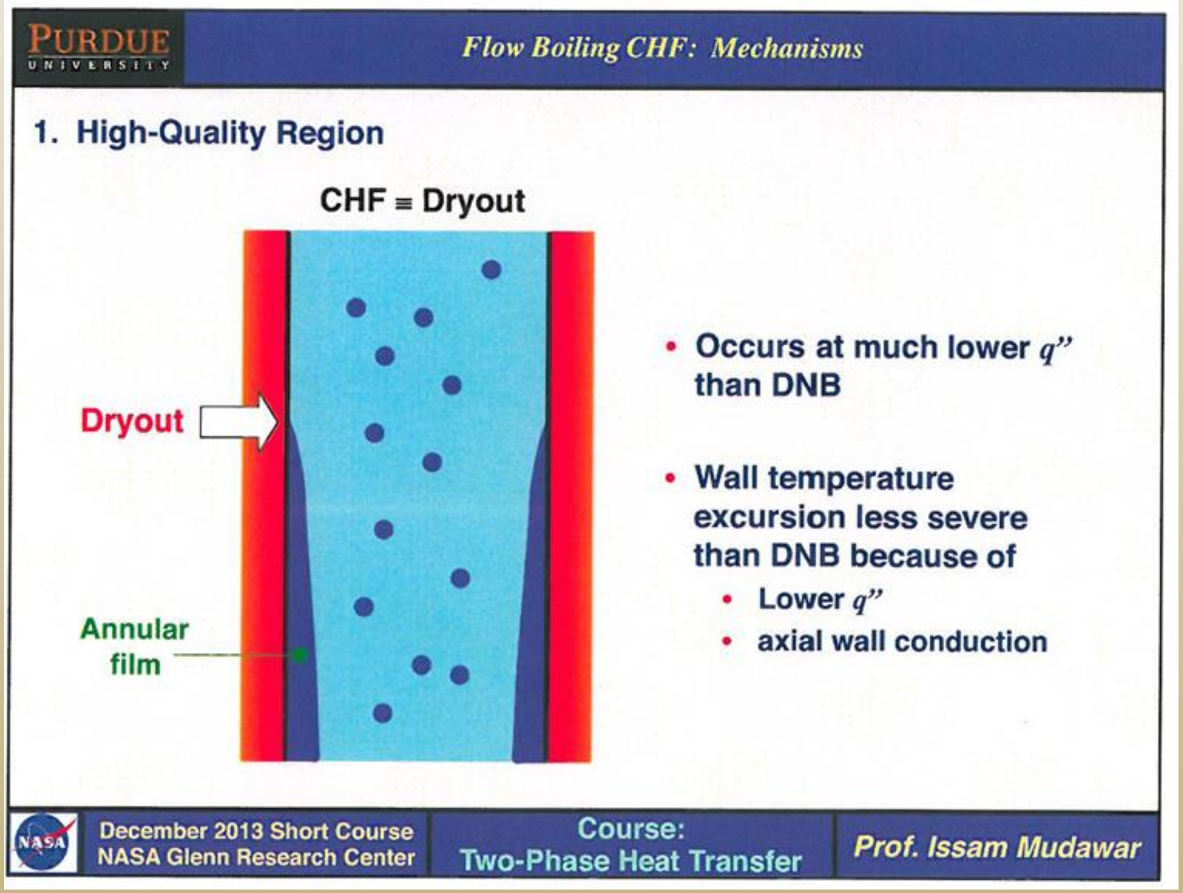
FLOW BOILING HEAT TRANSFER

- Flow Boiling Critical Heat Flux (CHF)



FLOW BOILING HEAT TRANSFER

- Flow Boiling Critical Heat Flux (CHF)



FLOW BOILING HEAT TRANSFER

Flow Boiling Critical Heat Flux (CHF)

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Interfacial Lift-off CHF Model

Use Separated Flow Model determine axial variations of:

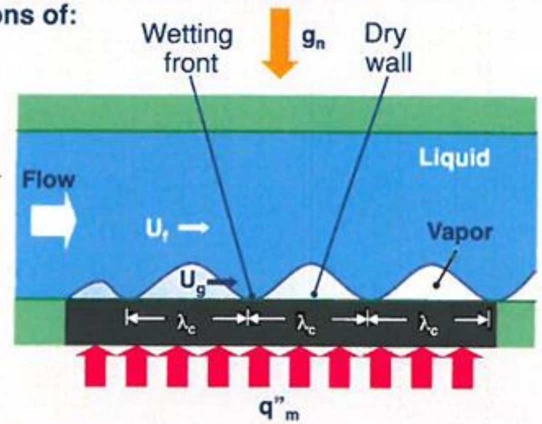
- U_g *vapor layer velocity*
- U_f *liquid layer velocity*
- δ *vapor layer thickness*

Critical interfacial wavelength

$$\frac{2\pi}{\lambda_c} = \frac{\rho''_f \rho''_g (U_g - U_f)^2}{2\sigma(\rho''_f + \rho''_g)} + \sqrt{\left[\frac{\rho''_f \rho''_g (U_g - U_f)^2}{2\sigma(\rho''_f + \rho''_g)}\right]^2 + \frac{(\rho_f - \rho_g) g_n}{\sigma}}$$

where $\rho''_f = \rho_f \coth(2\pi H_f / \lambda_c)$

$\rho''_g = \rho_g \coth(2\pi H_g / \lambda_c)$



Mean pressure difference across wetting front

$$\overline{P_f - P_g} = \frac{4\pi\sigma\delta}{b\lambda_c^2} \sin(\pi b)$$

where b is ratio of wetting front length to wavelength

Interfacial lift-off criterion

$$\overline{P_f - P_g} = \rho_g \left[\frac{q''_w}{\rho_g (h_{fg} + c_{p,f} \Delta T_{sub,o})}\right]^2$$

Surface energy balance

$$q''_m = b q''_w$$

Critical Heat Flux for slight subcooled inlet conditions

$$\boxed{q''_m = b \rho_g (h_{fg} + c_{p,f} \Delta T_{sub,o}) \left[\frac{4\pi\sigma\delta}{\rho_g b \lambda_c^2} \sin(\pi b)\right]^{1/2}}$$

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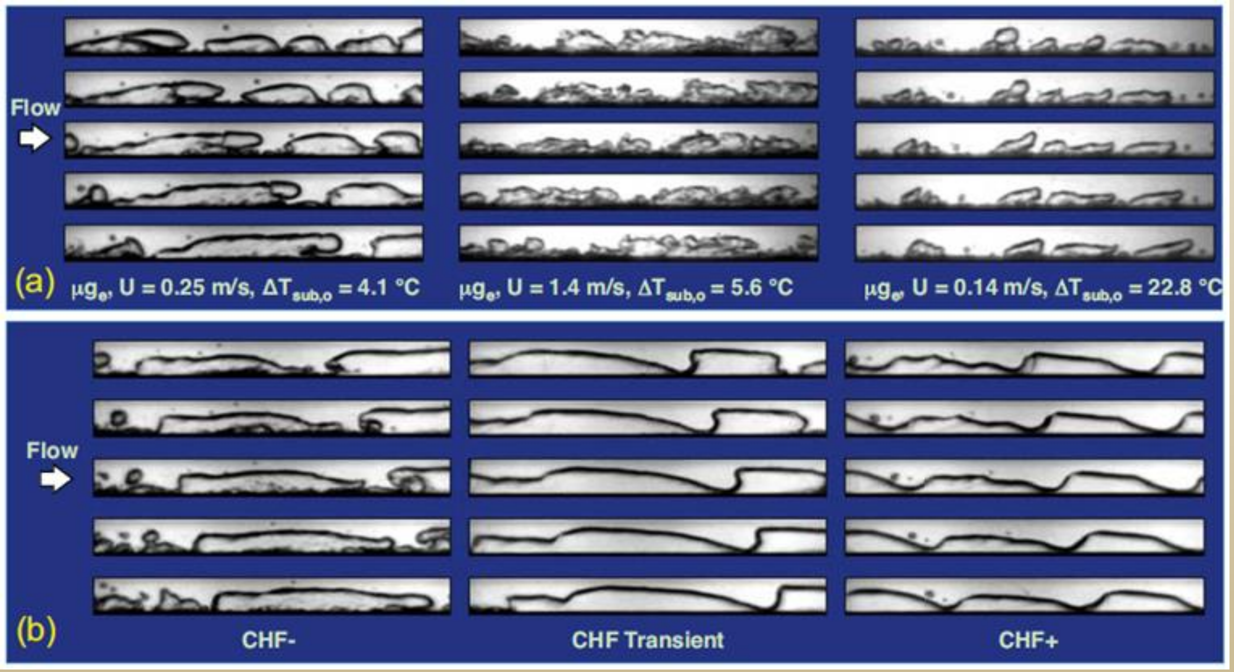
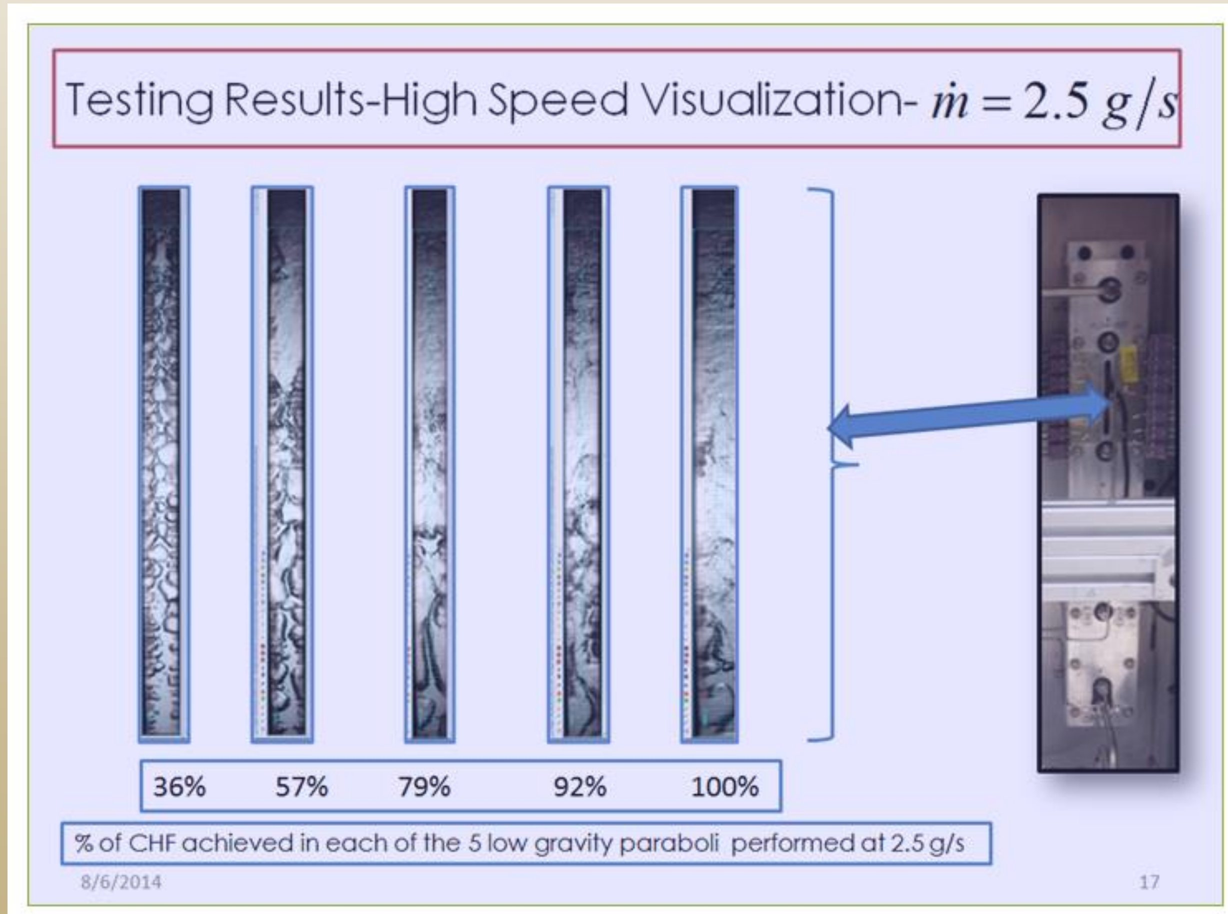
FLOW BOILING HEAT TRANSFER

- Flow Boiling Critical Heat Flux (CHF)



FLOW BOILING HEAT TRANSFER

- Flow Boiling Critical Heat Flux (CHF)



FLOW BOILING HEAT TRANSFER

Flow Boiling Critical Heat Flux (CHF)

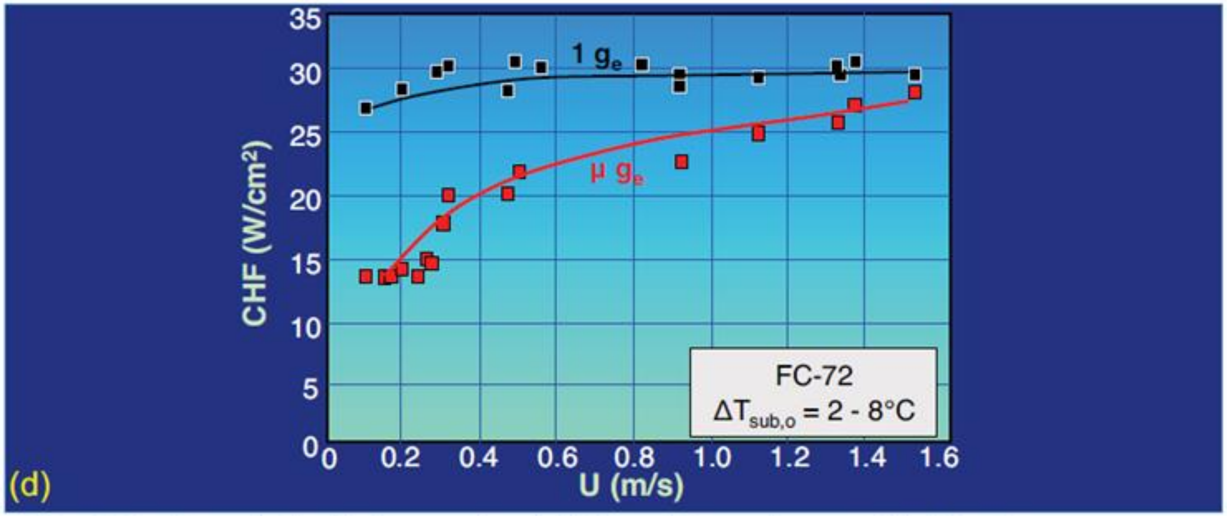


Fig. 4.2 Parabolic flight results: (a) Wavy Vapor Layer CHF Regime prevalent in μg_e at both low and high velocities as well as near-saturated and subcooled conditions. (b) CHF transient in μg_e for U = 0.15 m/s and $\Delta T_{sub,o}$ = 3.0°C. (c) Pool-boiling-like flow boiling at 1.8 g_e. (d) CHF data for μg_e and horizontal 1 g_e flow boiling.

$$\frac{Bo}{We^2} = \frac{(\rho_f - \rho_g)(\rho_f + \rho_g)^2 \sigma g_e}{\rho_f^2 \rho_g^2 U^4} \le 0.09.$$

$$\frac{1}{Fr} = \frac{(\rho_f - \rho_g) g_e D_h}{\rho_f U^2} \le 0.13.$$

FLOW BOILING HEAT TRANSFER

- Examples of Flow Boiling Critical Heat Flux Prediction

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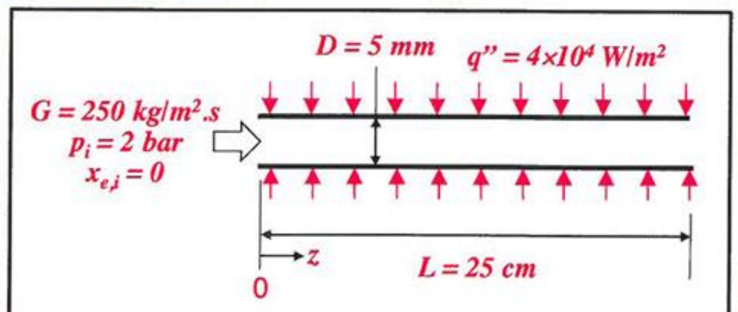
Numerical Example 5: Determination of Dryout Incipience Quality using Kim & Mudawar (2013) Correlation and CHF using Katto (1981) Correlation for Flow Boiling in Tubes

Saturated FC-72 ($x_e = 0$) at mass velocity of G = 250 kg/m².s and inlet pressure of p_i = 2 bar enters a horizontal circular tube of diameter D = 5 mm and length L = 25 cm, where it is subjected to a constant heat flux of q'' = 4×10⁴ W/m².

Assuming constant thermophysical properties and using five Δz increments along the flow direction, determine the following:

(a) Dryout incipience quality, x_{di}, using the Kim & Mudawar (2013) correlation

(b) Critical heat flux using Katto (1981) correlation



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FLOW BOILING HEAT TRANSFER

Flow Boiling CTH

PURDUE UNIVERSITY — *Numerical Example 5: Determination of Dryout Incipience Quality using Kim & Mudawar (2013) Correlation and CHF using Katto (1981) Correlation for Flow Boiling in Tubes*

Solution:

Thermophysical properties of FC-72 at p = 2 bar: $c_{p,f}$ = 1136 J/kg.K, h_{fg} = 87272 J/kg, v_f = 0.0006515 m³/kg, v_g = 0.0387 m³/kg, μ_f = 349.0×10⁻⁶ kg/m.s, μ_g = 12.3×10⁻⁶ kg/m.s, σ = 0.0062 N/m, k_f = 0.0514 W/m.K, Pr_f = 7.7212, p_{crit} = 1830 kPa

(a) First, determine quality at the exit ($z = L$):

$$x_{e,L} = \frac{(\pi D)\,q''}{G\left(\frac{\pi D^2}{4}\right)h_{fg}}L = \frac{(\pi\times 0.005\ m)\times 4\times 10^4\ W/m^2}{250\ kg/m^2s\times\left[\frac{\pi\times(0.005\ m)^2}{4}\right]\times 87272\ J/kg}\times(0.25\ m) = \boxed{0.367}$$

Dryout incipience quality (Kim and Mudawar, 2013):

$$x_{di} = 1.4\,We_{fo}^{0.03}P_R^{0.08} - 15.0\left(Bo\frac{P_H}{P_F}\right)^{0.15}Ca^{0.35}\left(\frac{\rho_g}{\rho_f}\right)^{0.06}$$

where $We_{fo} = \dfrac{G^2D}{\rho_f\sigma}$, $P_R = \dfrac{P}{P_{crit}}$, $Bo = \dfrac{q''_H}{G\,h_{fg}}$, $Ca = \dfrac{\mu_f G}{\rho_f\sigma}$

$$x_{di} = 1.4\times\left[\frac{(250\ kg/m^2s)^2\times 0.005\ m}{(0.0006515\ m^3/kg)^{-1}\times 0.0062\ N/m}\right]^{0.03}\times\left(\frac{200\ kPa}{1830\ kPa}\right)^{0.08}$$
$$-15.0\times\left(\frac{4\times10^4\ W/m^2}{250\ kg/m^2s\times 87272\ J/kg}\times 1\right)^{0.15}\times\left[\frac{349.0\times10^{-6}\ kg/ms\times 250\ kg/m^2s}{(0.0006515\ m^3/kg)^{-1}\times 0.0062\ N/m}\right]^{0.35}\times\left(\frac{0.0006515\ m^3/kg}{0.0387\ m^3/kg}\right)^{0.06}$$

$$x_{di} = \boxed{0.419}$$

Since x_{di} (= 0.419) is larger than $x_{e,L}$ (= 0.367), dryout is not expected anywhere along the tube.

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Flow Boiling CTH

Numerical Example 5: Determination of Dryout Incipience Quality using Kim & Mudawar (2013) Correlation and CHF using Katto (1981) Correlation for Flow Boiling in Tubes

(b) Critical heat flux (Katto, 1981):

$$CHF = q''_m = q''_{m0}\left(1.0 + K\frac{\Delta h_{sub,in}}{h_{fg}}\right)$$

$$q''_{m01} = 0.25\left(G h_{fg}\right)\frac{1}{L/D}$$

$$q''_{m02} = C\left(G h_{fg}\right) We^{-0.043}\frac{1}{L/D} \quad \text{where} \quad We = \frac{G^2 L}{\sigma \rho_f}$$

$$q''_{m03} = 0.15\left(G h_{fg}\right)\left(\frac{\rho_g}{\rho_f}\right)^{0.133} We^{-1/3}\frac{1}{1+0.0077\,L/D}$$

$$q''_{m04} = 0.26\left(G h_{fg}\right)\left(\frac{\rho_g}{\rho_f}\right)^{0.133} We^{-0.433}\frac{\left(L/D_H\right)^{0.171}}{1+0.0077\,L/D}$$

$$K_1 = 1$$

$$K_2 = \frac{0.261}{C\,We^{-0.043}}$$

$$K_3 = \frac{0.5556\left(0.0308 + D/L\right)}{\left(\rho_g/\rho_f\right)^{0.133} We^{-1/3}}$$

FLOW BOILING HEAT TRANSFER

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Numerical Example 5: Determination of Dryout Incipience Quality using Kim & Mudawar (2013) Correlation and CHF using Katto (1981) Correlation for Flow Boiling in Tubes

For $L/D < 50,\ C = 0.25$

For $L/D > 50,\ C = 0.34$

When $q'_{m01} < q'_{m02},\ q'_{m0} = q'_{m01},\ K = K_1$

When $q'_{m01} > q'_{m02},\ if\ q'_{m02} < q'_{m03},\ q'_{m0} = q'_{m02},\ K = K_2$

If $q'_{m02} > q'_{m03},\ if\ q'_{m03} < q'_{m04},\ q'_{m0} = q'_{m03},\ K = K_3$

If $q'_{m03} > q'_{m04},\ q'_{m0} = q'_{m04}$

$q'_{m01} = 109{,}090\ W/m^2$

$q'_{m02} = 107{,}910\ W/m^2$

$q'_{m03} = 116{,}330\ W/m^2$

$q'_{m04} = 188{,}190\ W/m^2$

$CHF = q'_m = \boxed{107{,}910\ W/m^2}$

Therefore, CHF will not occur within the tube because the wall heat flux ($q'' = 4\times10^4$ W/m^2) is small than critical heat flux (CHF = 10.8×10^4 W/m^2).

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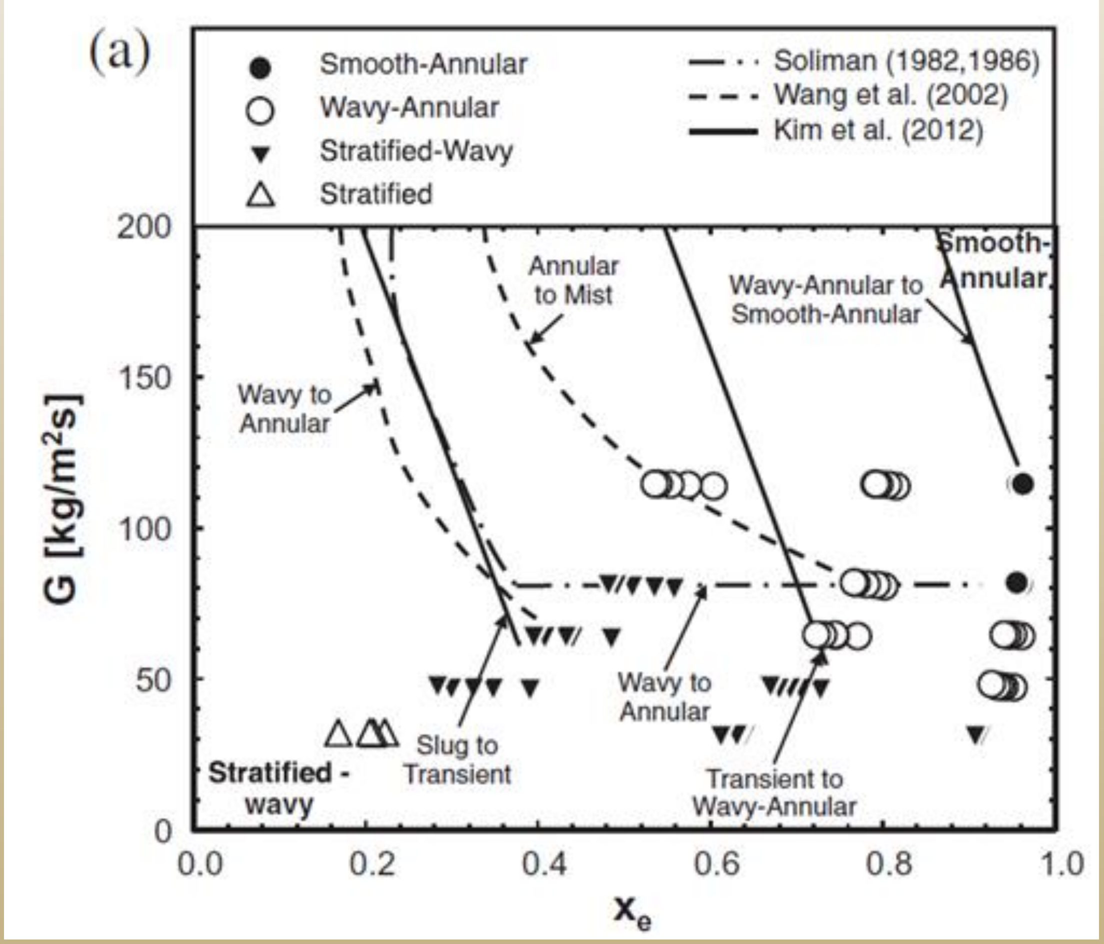
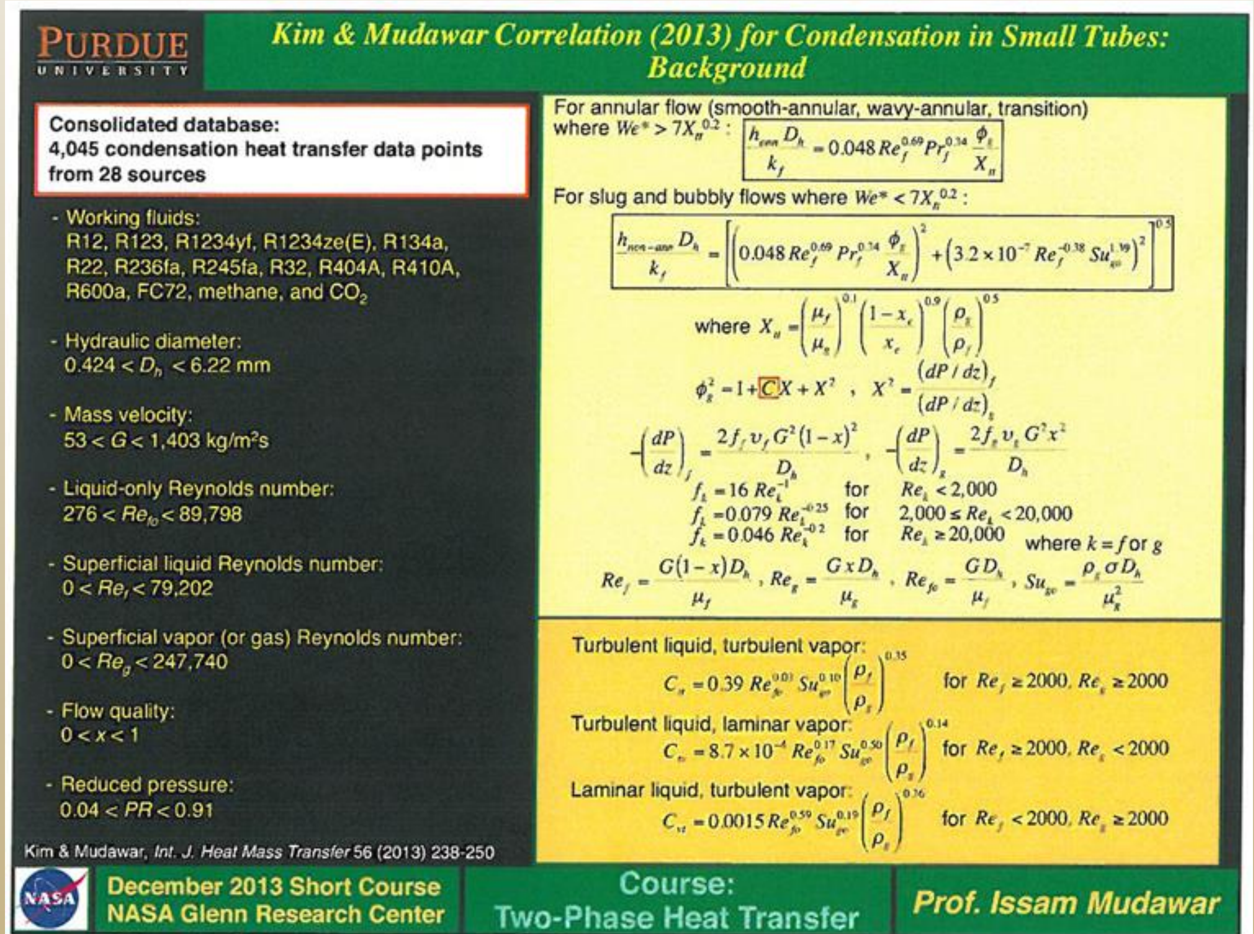
FLOW CONDENSATION HEAT TRANSFER

- Heat Transfer Coefficients for Condensation in Tubes



FLOW CONDENSATION HEAT TRANSFER

- Flow Condensation Heat Transfer



FLOW CONDENSATION HEAT TRANSFER

Flow Condensation Heat Transfer

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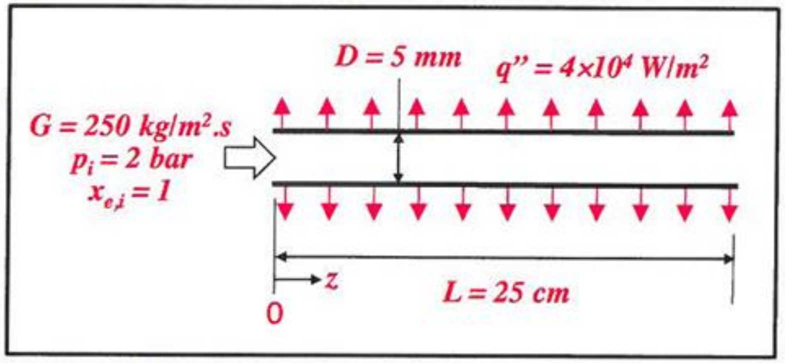
Numerical Example 1: Determination of Condensation Heat Transfer Coefficient using Kim and Mudawar (2013) Correlation

Saturated FC-72 ($x_e = 1$) at mass velocity of G = 250 kg/m².s and inlet pressure of p_i = 2 bar enters a horizontal circular tube of diameter D = 5 mm and length L = 25 cm, where it is subjected to constant heat rejection at q'' = 4×10⁴ W/m².
Assuming constant thermophysical properties and using five Δz increments along the flow direction, use the Kim and Mudawar (2013) correlation to determine the following:

(a) $x_e(z), x_{e,L}$

(b) $h(z)$

(c) $\overline{h}(z)$



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- Flow Condensation Heat Transfer

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Numerical Example 1: Determination of Condensation Heat Transfer Coefficient using Kim and Mudawar (2013) Correlation

Solution:

Thermophysical properties of FC-72 at p = 2 bar: $c_{p,f}$ = 1136 J/kg.K, h_{fg} = 87272 J/kg, v_f = 0.0006515 m³/kg, v_g = 0.0387 m³/kg, μ_f = 349.0x10⁻⁶ kg/m.s, μ_g = 12.3x10⁻⁶ kg/m.s, σ = 0.0062 N/m, k_f = 0.0514 W/m.K, Pr_f = 7.7212, p_{crit} = 1830 kPa

(a)

$$x_e = 1 - \frac{P_H}{W\, h_{fg}} \int_0^z q''\, dz = 1 - \frac{(\pi D)\, q''}{G\left(\frac{\pi D^2}{4}\right) h_{fg}} z = 1 - \frac{(\pi \times 0.005\ m) \times 4 \times 10^4\ W/m^2}{250\ kg/m^2 s \times \left[\frac{\pi \times (0.005\ m)^2}{4}\right] \times 87272\ J/kg} z = \boxed{1 - 1.467 z}$$

$$x_{e,L} = 1 - 1.467 \frac{1}{m} \times 0.25 m = \boxed{0.633}$$

(b) For annular flow (smooth-annular, wavy-annular, transition) where $We^* > 7X_{tt}^{0.2}$:

$$\frac{h_{ann} D}{k_f} = 0.048\, Re_f^{0.69}\, Pr_f^{0.34} \frac{\phi_g}{X_{tt}}$$

For slug and bubbly flows where $We^* < 7X_{tt}^{0.2}$:

$$\frac{h_{non-ann} D}{k_f} = \left[\left(0.048\, Re_f^{0.69}\, Pr_f^{0.34} \frac{\phi_g}{X_{tt}}\right)^2 + \left(3.2 \times 10^{-7} Re_f^{-0.38} Su_{go}^{1.39}\right)^2\right]^{0.5}$$

where

$$X_{tt} = \left(\frac{\mu_f}{\mu_g}\right)^{0.1} \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_g}{\rho_f}\right)^{0.5}, \quad \phi_g^2 = 1 + C X + X^2, \quad X^2 = \frac{(dP/dz)_f}{(dP/dz)_g}$$

$$-\left(\frac{dP}{dz}\right)_f = \frac{2 f_f v_f G^2 (1-x_e)^2}{D}, \quad -\left(\frac{dP}{dz}\right)_g = \frac{2 f_g v_g G^2 x_e^2}{D}$$

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FLOW CONDENSATION HEAT TRANSFER

- Flow Condensation Heat Transfer

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Numerical Example 1: Determination of Condensation Heat Transfer Coefficient using Kim and Mudawar (2013) Correlation

$$f_k = 16\,Re_k^{-1} \quad \text{for} \quad Re_k < 2{,}000$$

$$f_k = 0.079\,Re_k^{-0.25} \quad \text{for} \quad 2{,}000 \le Re_k < 20{,}000$$

$$f_k = 0.046\,Re_k^{-0.2} \quad \text{for} \quad Re_k \ge 20{,}000$$

$$C_{tt} = 0.39\,Re_{fo}^{0.03}\,Su_{go}^{0.10}\left(\frac{\rho_f}{\rho_g}\right)^{0.35} \quad , \quad C_{vt} = 0.0015\,Re_{fo}^{0.59}\,Su_{go}^{0.19}\left(\frac{\rho_f}{\rho_g}\right)^{0.36}$$

$$C_{tv} = 8.7\times10^{-4}\,Re_{fo}^{0.17}\,Su_{go}^{0.50}\left(\frac{\rho_f}{\rho_g}\right)^{0.14} \quad , \quad C_{vv} = 3.5\times10^{-5}\,Re_{fo}^{0.44}\,Su_{go}^{0.50}\left(\frac{\rho_f}{\rho_g}\right)^{0.48}$$

$$We^* = 2.45\,\frac{Re_g^{0.64}}{Su_{go}^{0.3}\left(1+1.09\,X_{tt}^{0.039}\right)^{0.4}} \quad \text{for} \quad Re_f \le 1250$$

$$We^* = 0.85\,\frac{Re_g^{0.79}\,X_{tt}^{0.157}}{Su_{go}^{0.3}\left(1+1.09\,X_{tt}^{0.039}\right)^{0.4}}\left[\left(\frac{\mu_g}{\mu_f}\right)^2\left(\frac{\upsilon_g}{\upsilon_f}\right)\right]^{0.084} \quad \text{for} \quad Re_f > 1250$$

where $k = f$ or g

$$Re_f = \frac{G(1-x_e)D}{\mu_f} \quad , \quad Re_g = \frac{G\,x_e\,D}{\mu_g} \quad , \quad Re_{fo} = \frac{G\,D}{\mu_f} \quad , \quad Su_{go} = \frac{\rho_g\,\sigma\,D}{\mu_g^2}$$

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Flow Condensation Heat Transfer

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Numerical Example 1: Determination of Condensation Heat Transfer Coefficient using Kim and Mudawar (2013) Correlation

Node #	1	2	3	4	5
x_e	0.927	0.853	0.780	0.707	0.633
$-(dp/dz)_f$	5.34	10.67	16.01	21.34	26.68
X	0.037	0.057	0.075	0.095	0.117
C	15.456	15.456	15.456	15.456	15.456
ϕ_g	1.255	1.370	1.472	1.573	1.680
Re_f	263	525	788	1051	1313
Su_{go}	5294700	5294700	5294700	5294700	5294700
X_{tt}	0.019	0.037	0.058	0.082	0.111
We^*	27.60	26.04	24.50	22.94	22.16
h	3135	2747	2500	2301	2124

(c) $\bar{h} = \frac{1}{L}\int_0^L h(z)\,dz = \boxed{2561\ W/m^2K}$

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- Course End